Tutorial GiBUU
Part A: Kinetic Theory and Implementation

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- Kinetic Theory and BUU equation

  GiBUU implementation

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Outline

- Part A: Kinetic Theory and GiBUU implementation
  - BUU equation
  - degrees of freedom
  - potentials
  - collision term
  - baryon-meson, baryon-baryon-collisions
  - Testparticles, Parallel vs. Full ensemble
  - Local collision criterion (beyond 2-particle collisions)
  - Initial state
    - Local Thomas Fermi vs. Readjusting
    - Frozen particles
- Part B: Hands-on
  - ...

Some kinetic theory

- **distribution function** $f(x,p) \quad x = (t, \vec{x}\,),\ p = (E, \vec{p}\,)$
  describes (density) distribution of (single) particles
- number of particles in a given phase-space volume:
  $$\Delta N = f(x,p)\, \Delta^3 x \Delta^3 p$$
- for each particle species: $f_N,\ f_\pi,\ f_\Delta,\ ,\ldots$
- **continuity equation** for free, non-interacting particles
  $$p^\mu \partial_\mu\, f(x,p) = 0$$
  straight line propagation of particles, no collisions
- adding external forces (mean field potentials): **Vlasov eq.**
  $$\left[\partial_t + (\nabla_p E)\nabla_r - (\nabla_r E)\nabla_p\right] f(x,p) = 0$$
  propagation through mean field, no collisions

Adding collisions

- forget about mean fields, but add collisions…
- continuity eq. + collision term → Boltzmann eq.

$$p^{\mu}\partial_{\mu} f(x,p) = C(x,p)$$

- collision integral has gain and loss term

$$C(x,p) = C_{\text{gain}}(x,p) + C_{\text{loss}}(x,p)$$

- mean fields and collision term:

Boltzmann-Uehling-Uhlenbeck eq. (BUU or VUU)

$$\left[\partial_t + (\nabla_p H_i)\nabla_r - (\nabla_r H_i)\nabla_p\right] f_i(\vec{r},t,\vec{p}) = C\left[f_i, f_j, \ldots\right]$$

The BUU equation

$$\left[\partial_t + (\nabla_p H_i)\nabla_r - (\nabla_r H_i)\nabla_p\right] f_i(\vec{r}, t, \vec{p}) = C\left[f_i, f_j, \ldots\right]$$

- describes space-time evolution of single particle densities
- index i represents particle species
  $\rightarrow$ one equation for each species

$$i = N, \Delta, \pi, \rho, \ldots$$

- Hamiltonian H_i
  - hadronic mean fields (Skyrme/Welke or RMF)
  - Coulomb
  - „off-shell-potential“
- collision term C
  - decay and scattering processes: 1-, 2- and 3-body
  - (low energy: resonance model, high energy: string model)
  - contains Pauli-blocking
- equations coupled via mean fields and via collision term

Degrees of Freedom

- GiBUU is purely hadronic (no partonic phase)
- leptons: usually not ‚transported‘, but
  - e+N, nu+N, gamma+N initial events
  - leptonic/photonic decays
- 61 baryons, 22 mesons
  (strangeness and charm included, no bottom)
- properties from Manley analysis (PDG for strange/charm)

- in principle one needs:
  - cross sections for collisions between all of them (all energies)
  - mean-field potentials for all species

  often not known, thus use hypothesis/models/guesses

Particle species

- important particles:

particle	mass	width	GiBUU ID	PDG IDs
N	0.983	0	1	p=2212, n=2112
Δ	1.232	0.118	2	2224, 2214, 2114, 1114
N^*			3-18	
Δ^*			19-31	
Λ	1.116	0	32	3122
Σ	1.189	0	33	3222,3212,3112
Λ^*, Σ^*			34-52	
π	0.138	0	101	$\pi^+ = 211$, $\pi^0 = 111$, $\pi^- = -211$
η	0.547		102	
ρ	0.775	0.149	103	213,113,-213
σ			104	
ω	0.782	0.004	105	
η'	0.957		106	
K	0.496	0	110	$K^+ = 321$, $K^0 = 311$
$\bar{K}$	0.496	0	111	$K^- = -321$, $\bar{K}^0 = -311$

https://gibuu.hepforge.org/trac/wiki/ParticleIDs

Mean-field potentials

- two types of mean-field potentials:
  - non-relativistic Skyrme-type potentials
  - relativistic mean fields (RMF)

- potential may enter single-particle energy as

$$H = \sqrt{(m+V)^2 + (\vec{p} + \vec{U}\,)^2} + U_0$$

- RMF is Lorentz vector U^μ
- Skyrme enters as U_0, bound to specific frame (LRF)

- Scalar Potential V: mass shift

Skyrme/Welke-like potential

$$U_0(x,\vec{p}) = A\frac{\rho}{\rho_0} + B\left(\frac{\rho}{\rho_0}\right)^{\gamma} + \frac{2C}{\rho_0}\sum_{i=p,n}\int\frac{g\mathrm{d}^3p'}{(2\pi)^3}\frac{f_i(x,\vec{p}\,')}{1+(\vec{p}-\vec{p}\,')^2/\Lambda^2} + d_{\mathrm{symm}}\frac{\rho_p(x)-\rho_n(x)}{\rho_0}\tau_i$$

$$\rho_0 = 0.168\,\mathrm{fm}^{-3}$$

- defined in local rest frame (LRF, baryon current vanishes)
- six parameters
- fixed to...
  - nuclear binding energy of 16 MeV at $\rho=\rho_0$ (iso-spin symm. matter)
  - nuclear-matter incompressibility K=200-380 MeV

Collision term

- contains one-, two-, and three-body collisions

$$C = C_{1\to X} + C_{2\to X} + C_{3\to X}$$

(1) resonance decays

(2) two-body collisions

- elastic and inelastic
- any number of particles in final state
- baryon-meson, baryon-baryon, meson-meson

(3) three-body collisions (only relevant at high densities)

- low energies: cross sections based on resonances

$$\text{e.g. } \pi N \to N^*, \; NN \to NN^*$$

- high energies: string fragmentation

Collision term

- 2-to-2 term $\quad (12 \leftrightarrow 1'2')$

$$
\begin{aligned}
&C^{(2,2)}(x,p_1)\\
&= C^{(2,2)}_{\mathrm{gain}}(x,p_1) - C^{(2,2)}_{\mathrm{loss}}(x,p_1)\\
&= \frac{\mathcal{S}_{1'2'}}{2p_1^0 g_{1'} g_{2'}} \int \frac{\mathrm{d}^4 p_2}{(2\pi)^4 2p_2^0} \int \frac{\mathrm{d}^4 p_{1'}}{(2\pi)^4 2p_{1'}^0} \int \frac{\mathrm{d}^4 p_{2'}}{(2\pi)^4 2p_{2'}^0}\\
&\quad \times (2\pi)^4 \delta^{(4)}\left(p_1 + p_2 - p_{1'} - p_{2'}\right) \overline{|\mathcal{M}_{12\to 1'2'}|^2}\\
&\quad \times \left[F_{1'}(x,p_{1'}) F_{2'}(x,p_{2'}) \overline{F}_1(x,p_1) \overline{F}_2(x,p_2)\right.\\
&\qquad \left. - F_1(x,p_1) F_2(x,p_2) \overline{F}_{1'}(x,p_{1'}) \overline{F}_{2'}(x,p_{2'})\right]
\end{aligned}
$$

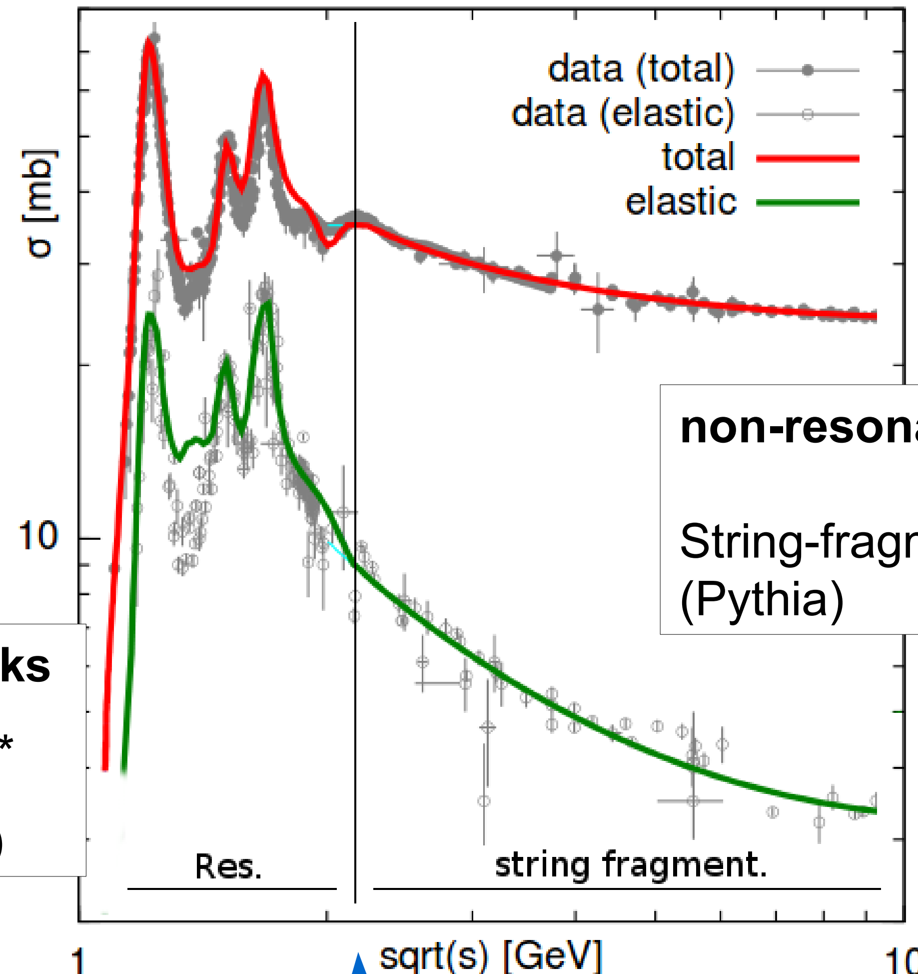
- $F(x,p) = 2\pi g\, f(x,p)\, \mathcal{A}(x,p)$

  $\overline{F}(x,p) = 2\pi g\, [1 - f(x,p)]\, \mathcal{A}(x,p)$

Pauli-blocking

Baryon-Meson collisions

- example: πN cross section



clear resonance peaks

excitation of N* and Δ*

(Breit-Wigner shapes)

non-resonant

String-fragmentation (Pythia)

$$\mathcal{A}(p) = \frac{1}{\pi} \frac{\sqrt{p^2}\Gamma}{(p^2 - M_0^2)^2 + p^2\Gamma^2}$$

$$\sqrt{s} = 2.2 \pm 0.2\,\mathrm{GeV}$$

Resonance Model

- resonance parameters, decays modes, widths:
  D.Manley, E.Saleski, PRD45 (1992) 4002
  PWA of $\pi N \to \pi N$ and $\pi N \to \pi\pi N$, consistency!!!

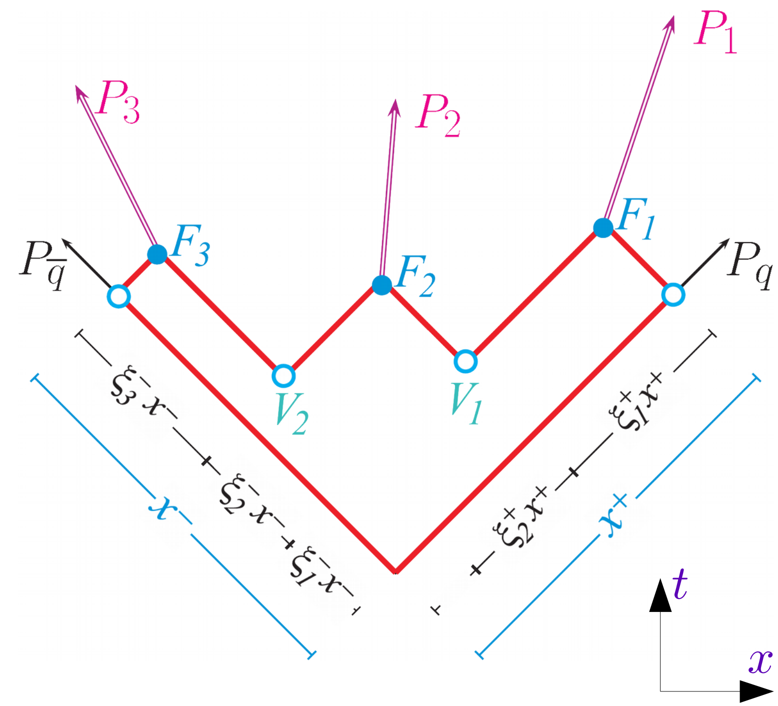
	rating	M_0 [MeV]	Γ_0 [MeV]	$\lvert\mathcal{M}^2\rvert/16\pi$ [mb GeV^2] NR	ΔR	branching ratio in % πN	ηN	$\pi\Delta$	ρN	σN	$\pi N^*(1440)$	$\sigma\Delta$
$P_{11}(1440)$	****	1462	391	70	—	69	—	22_P	—	9	—	—
$S_{11}(1535)$	***	1534	151	8	60	51	43	—	$2_S + 1_D$	1	2	—
$S_{11}(1650)$	****	1659	173	4	12	89	3	2_D	3_D	2	1	—
$D_{13}(1520)$	****	1524	124	4	12	59	—	$5_S + 15_D$	21_S	—	—	—
$D_{15}(1675)$	****	1676	159	17	—	47	—	53_D	—	—	—	—
$P_{13}(1720)$	*	1717	383	4	12	13	—	—	87_P	—	—	—
$F_{15}(1680)$	****	1684	139	4	12	70	—	$10_P + 1_F$	$5_P + 2_F$	12	—	—
$P_{33}(1232)$	****	1232	118	OBE	210	100	—	—	—	—	—	—
$S_{31}(1620)$	**	1672	154	7	21	9	—	62_D	$25_S + 4_D$	—	—	—
$D_{33}(1700)$	*	1762	599	7	21	14	—	$74_S + 4_D$	8_S	—	—	—
$P_{31}(1910)$	****	1882	239	14	—	23	—	—	—	—	67	10_P
$P_{33}(1600)$	***	1706	430	14	—	12	—	68_P	—	—	20	—
$F_{35}(1905)$	***	1881	327	7	21	12	—	1_P	87_P	—	—	—
$F_{37}(1950)$	****	1945	300	14	—	38	—	18_F	—	—	—	44_F

$$\Gamma_{R\to ab}(m) = \Gamma^0_{R\to ab}\frac{\rho_{ab}(m)}{\rho_{ab}(M^0)}$$

$$\rho_{ab}(m) = \int p_a^2 p_b^2 \mathcal{A}_a(p_a^2)\mathcal{A}_b(p_b^2)\frac{p_{ab}}{m} B^2_{L_{ab}}(p_{ab}R)\mathcal{F}^2_{ab}(m)$$

(Lund) String-fragmentation (PYTHIA)

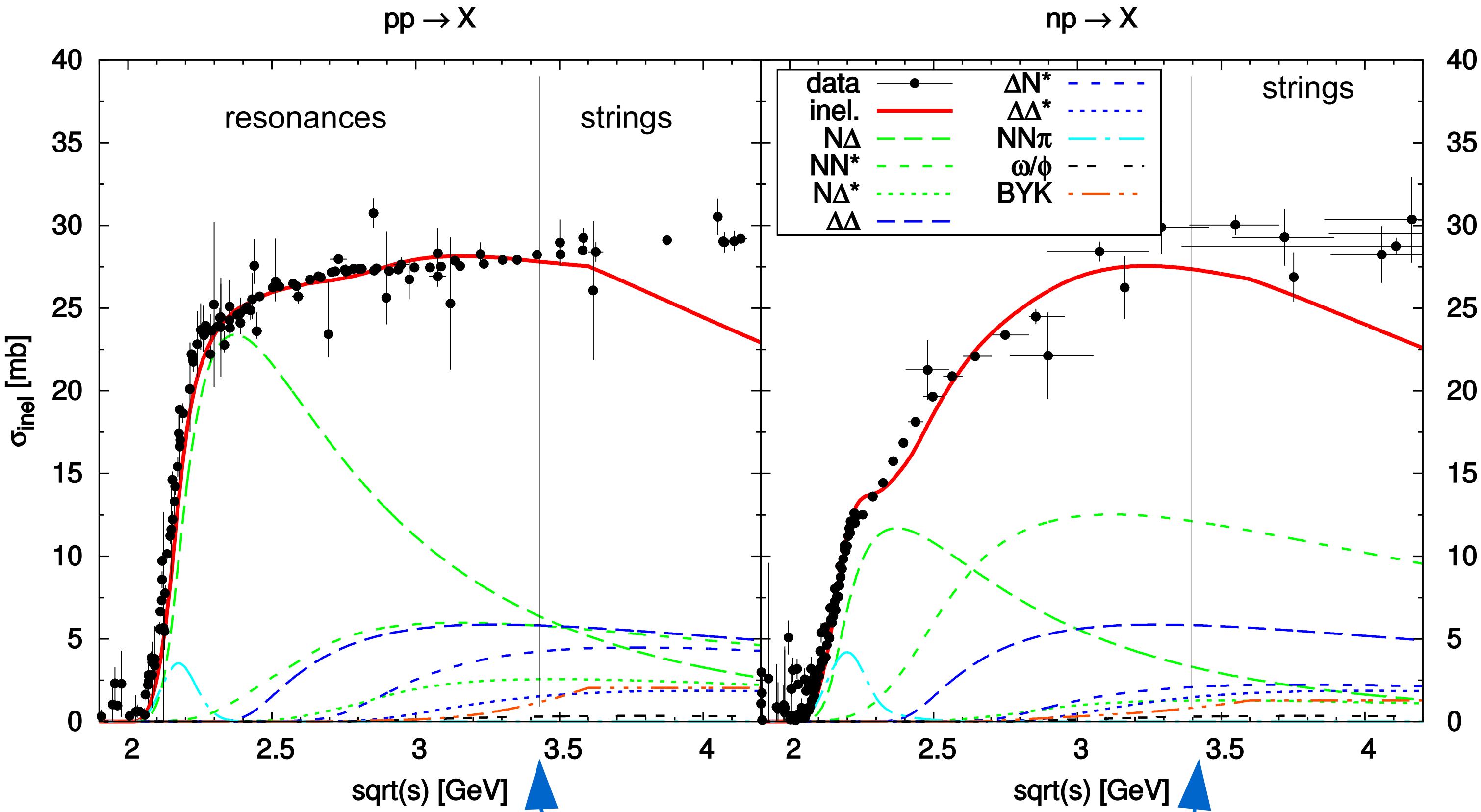
- *idea:*
  hard qq scattering (pQCD)
  creates a color flux tube ('string')
  which then fragments into hadrons
  (via $q\bar{q}$ pair production)



- high energy: 10 GeV...
- "Lund string model"
  implementation: PYTHIA (Jetset)
- only low-lying resonances
- phenomenological fragmentation function
  (when and how does a string break?)
- parameters fitted to data (different 'tunes' available)

Baryon-Baryon Collisions

- low energy: resonance model, high energy: string model
- no nice peaks due to two-body kinematics
- *NN→NR,ΔR* (*R*=Δ,*N**,Δ*)



$$\sqrt{s} = 3.4 \pm 0.1\,\mathrm{GeV}$$

Testparticle ansatz

$$\left[\partial_t + (\nabla_p H_i)\nabla_r - (\nabla_r H_i)\nabla_p\right] f_i(\vec{r},t,\vec{p}) = C\left[f_i, f_j, \ldots\right]$$

- *idea:*
  approximate full phase-space density distribution by a sum of delta-functions

$$f\left(\vec{r},t,\vec{p}\right) \sim \sum_{i=1}^{N_{\text{test}}} \delta\left(\vec{r}-\vec{r}_i(t)\right)\delta\left(\vec{p}-\vec{p}_i(t)\right)$$

- each delta-function represents one (test-)particle with a sharp position and momentum

- large number of test particles needed

Ensemble techniques

- **"full ensembles" technique**
- every testparticle may interact with every other one
- rescaling of cross section

$$\sigma_{ij} \to \frac{1}{N_{\mathrm{test}}} \sigma_{ij}$$

- Pros:
  - locality of collisions
- Cons:
  - calculational time: collisions scale with $(N_{\mathrm{test}})^2$
  - energy not conserved per ensemble, on average only
  - conserved quantum numbers are respected on average only ('canonical')

$$\underbrace{K}_{\text{ensemble } i} + \underbrace{\overline{K}}_{\text{ensemble } j} \to \pi\pi$$

Ensemble techniques

- **"parallel ensembles" technique**
- *idea:*

  testparticle index is also ensemble index

- $N_{\rm test}$ independent runs, densities etc. may be averaged

- Pros:
  - calculational time: collisions scale with $N_{\rm test}$
  - conserved quantum numbers are strictly respected ('microcanonical')
- Cons:
  - non-locality of collisions $\sigma_{ij} \simeq 30\,\mathrm{mb} \;\rightarrow\; r = 1\,\mathrm{fm}$

Time evolution

- time axis is discretized
  - collisions only happen at discrete time steps,
  - between collisions: propagation (through mean fields)

- typical time-step size: $\Delta t = 0.1\text{-}0.2\,\mathrm{fm}/c$
- start at $t{=}0$ and run N timesteps until $t_{\max}$
- typically:

$$N\,\Delta t = t_{\max} \approx 20\text{-}50\,\mathrm{fm}/c$$

$$\Longrightarrow\ N \approx 100\text{-}1000$$

- density/potentials: if not analytically, recalc at every step

Cross section: Geometric interpretation

- particle i and particle j collide, if during timestep Δt

$$r_{ij}(t) = |\vec{r}_i(t) - \vec{r}_j(t)| \overset{!}{\leq} \frac{\sqrt{\sigma_{ij}}}{\pi}$$

- problem 1: only for 2-body collisions

- problem 2: not invariant under Lorentz-Trafos
  - different frames may lead to different ordering of collisions
  - specific frame ('calculational frame') needed

Cross section: Stochastic interpretation

- collision rate per unit phase space

massless, no $(2\pi)^3$

$$\frac{\Delta N_{\rm coll}^{2\to 2}}{\Delta t \Delta^3 x \Delta^3 p_1} = \frac{\Delta^3 p_2}{2E_1\, 2E_2} f_1 f_2 \int \frac{\mathrm{d}^3 p_1'}{2E_1'} \frac{\mathrm{d}^3 p_2'}{2E_2'} |\mathcal{M}| \delta^{(4)}(p_1 + p_2 - p_1' - p_2')$$

$$\sigma_{22} = \frac{1}{2s} \int \frac{\mathrm{d}^3 p_1'}{2E_1'} \frac{\mathrm{d}^3 p_2'}{2E_2'} |\mathcal{M}| \delta^{(4)}(p_1 + p_2 - p_1' - p_2')$$

$$f_i = \frac{\Delta N_i}{\Delta^3 x \Delta^3 p}$$

- collision probability in unit box $\Delta^3 x$ and unit time Δt

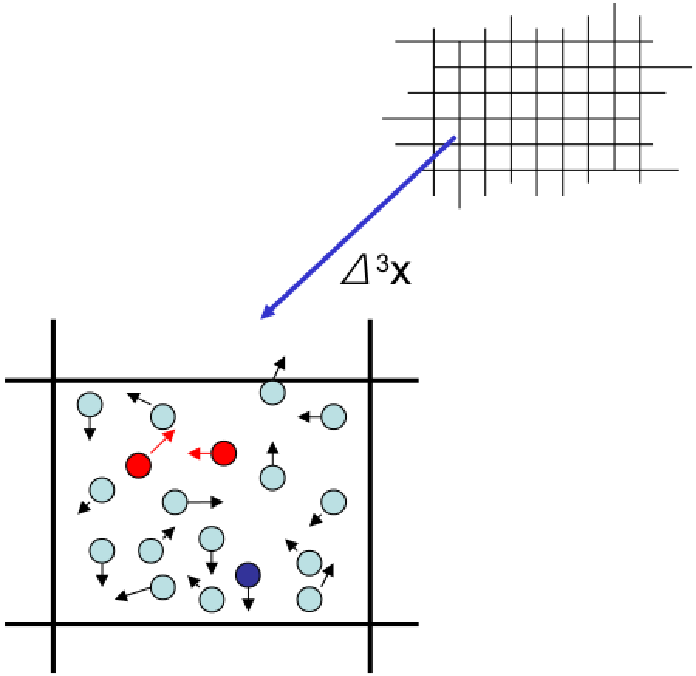
$$P_{22} = \frac{\Delta N_{\rm coll}^{2\to 2}}{\Delta N_1 \Delta N_2} = v_{\rm rel} \sigma_{22} \frac{\Delta t}{\Delta^3 x} \qquad \left(v_{\rm rel} = \frac{s}{2E_1 E_2} \right)$$

- generalisable to n-body collisions

Cross section: Stochastic interpretation

- discretize **time** and **space**

$$P_{2\to X} = v_{\rm rel}\sigma_{2\to X}\frac{\Delta t}{\Delta V}$$
$$P_{3\to X} = \frac{I_{3\to X}}{8E_1E_2E_3}\frac{\Delta t}{(\Delta V)^2}$$



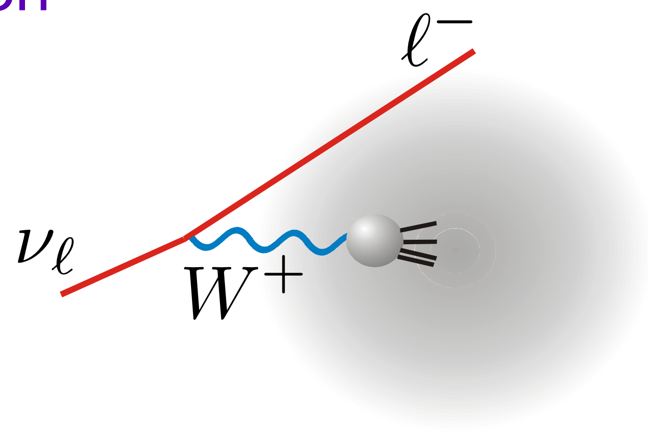
- together with 'full ensemble'
- n particles in cell, randomly select $n/2$ pairs

$$P_2 \to \frac{n(n-1)/2}{n/2}P_2$$

- calculational time: collisions scale approx. with $N_{\rm test}$
- labeled as "local ensemble method"

Nuclear Reactions

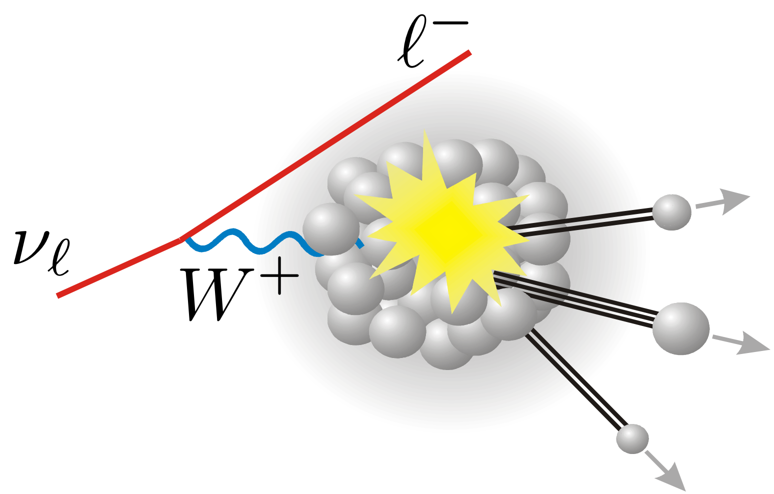
- elementary interaction on nucleon



additional:

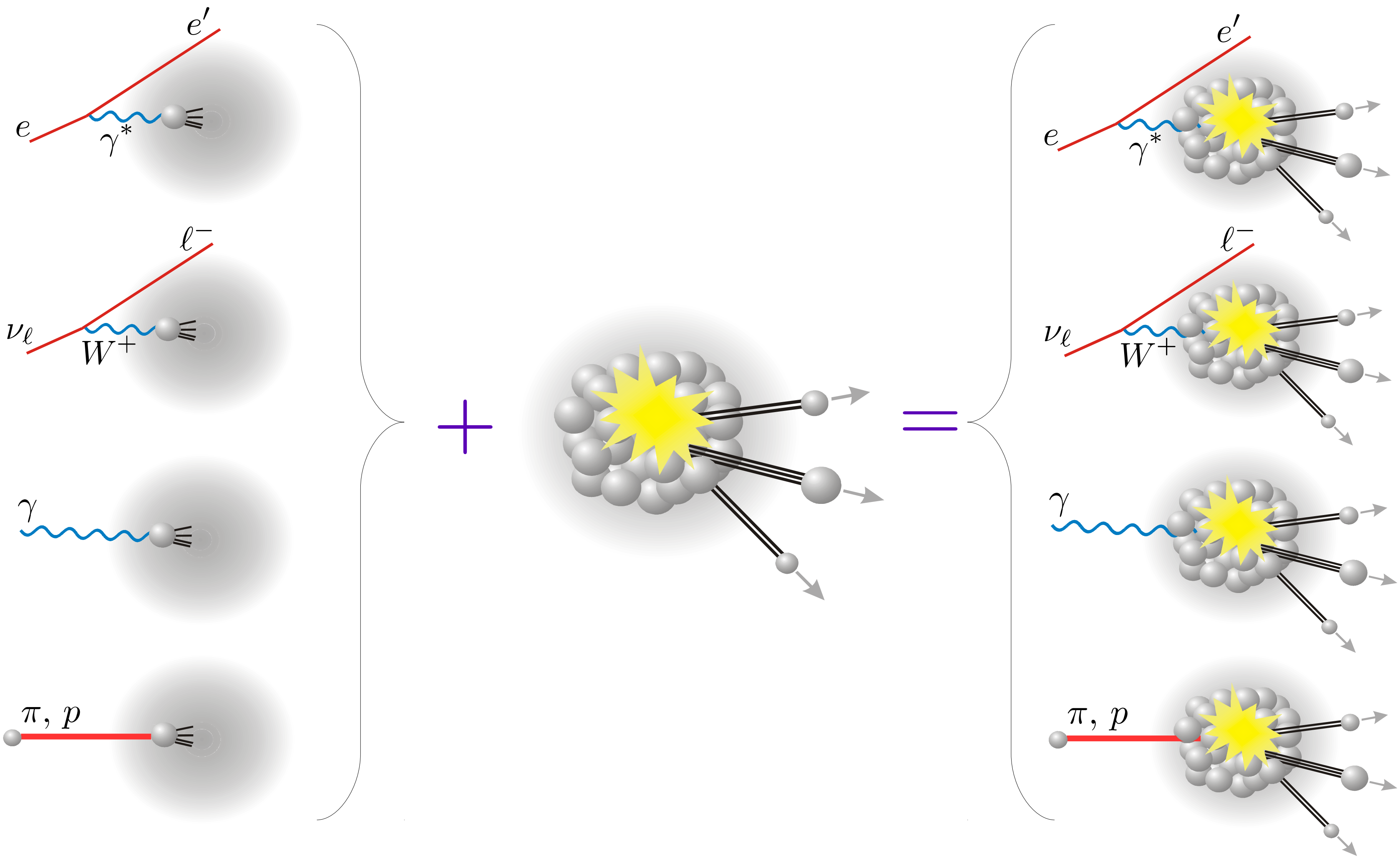
  - binding energies
  - Fermi motion
  - Pauli blocking
  - (coherence length effects)

- propagation of final state



  - elastic/inelastic scatterings
  - mean fields

GiBUU = plug-in system



init + FSI = full event

Nuclear ground state

- density distribution: Woods-Saxon (or harm. Oscillator)
- particle momenta: ‘Local Thomas-Fermi approximation’

$$f_{(\mathrm{n,p})}(\vec{r},\vec{p}) = \Theta\left[p_{F(\mathrm{n,p})}(\vec{r}) - |\vec{p}|\right]$$

- Fermi-momentum:

$$p_{F(\mathrm{n,p})}(\vec{r}) = \left(3\pi^2\rho_{(\mathrm{n,p})}(\vec{r})\right)^{1/3}$$

- Fermi-energy:

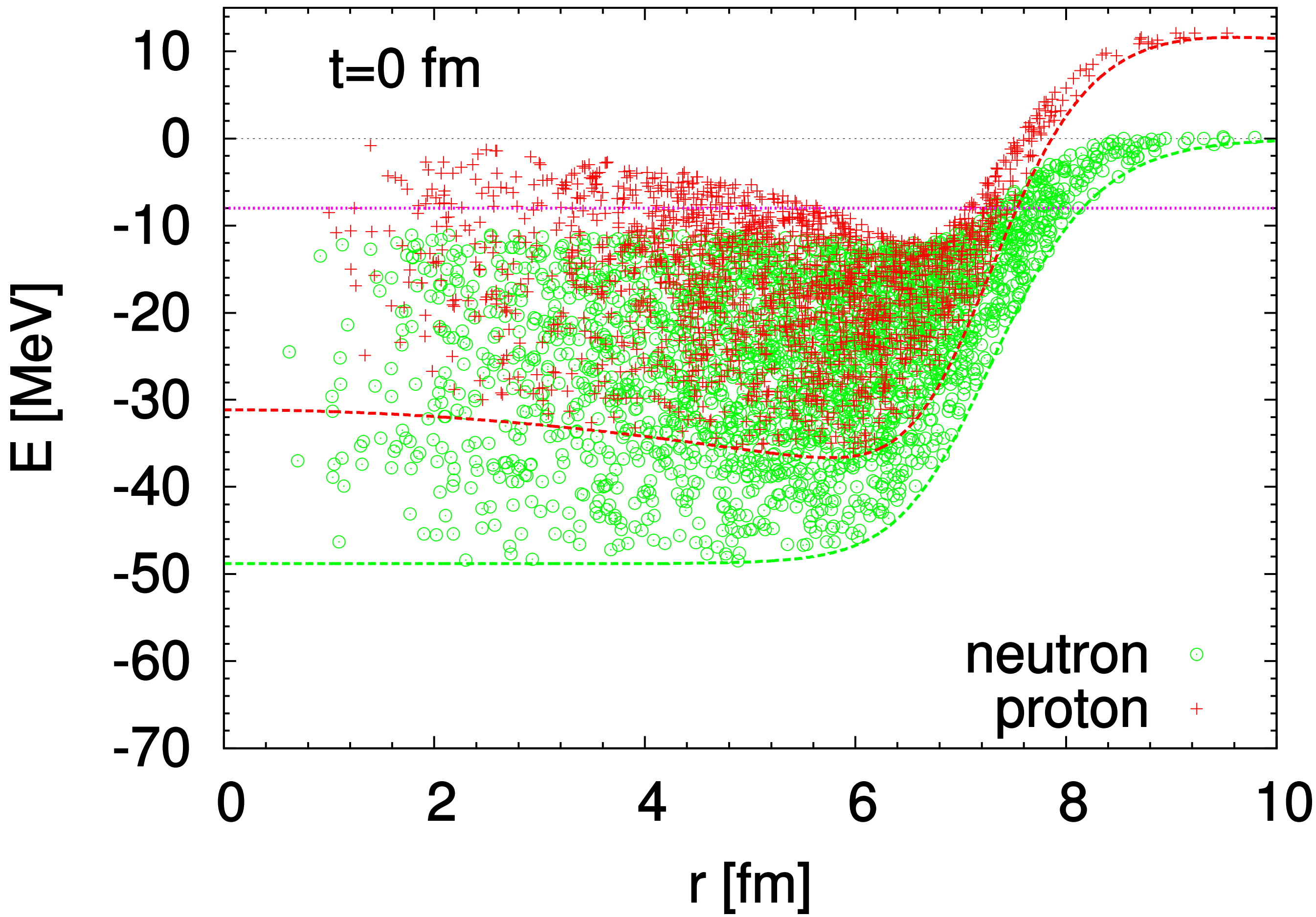
$$E_{F(\mathrm{n,p})} = \sqrt{p^2_{F(\mathrm{n,p})} + m_N^2} + U_{(\mathrm{n,p})}(\vec{r}, p_F)$$

potential: see above

Nuclear ground state

- LTF: time evolution en detail

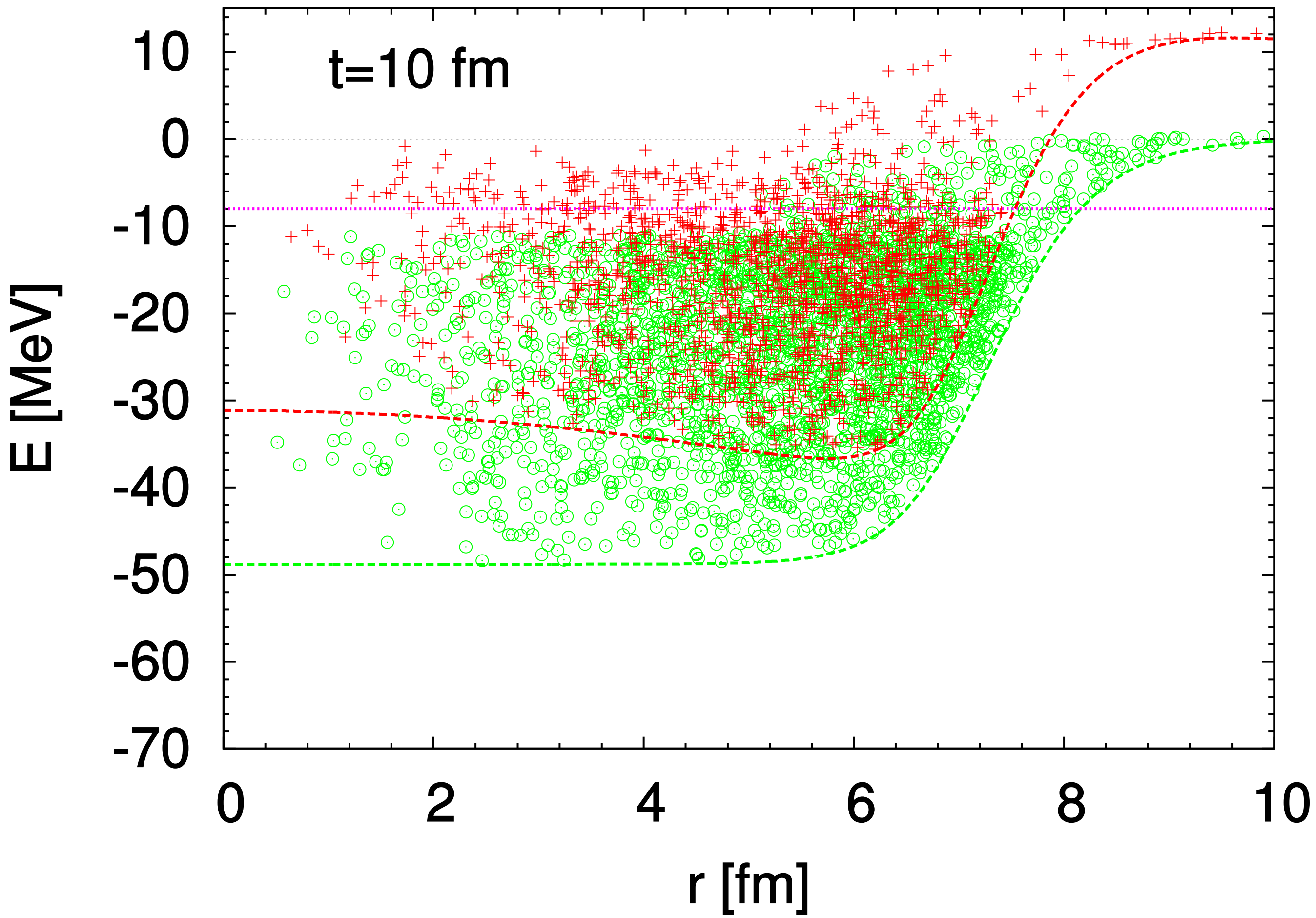
non-mom.dep potential, asymmetry-term, Coulomb



Nuclear ground state

- LTF: time evolution en detail

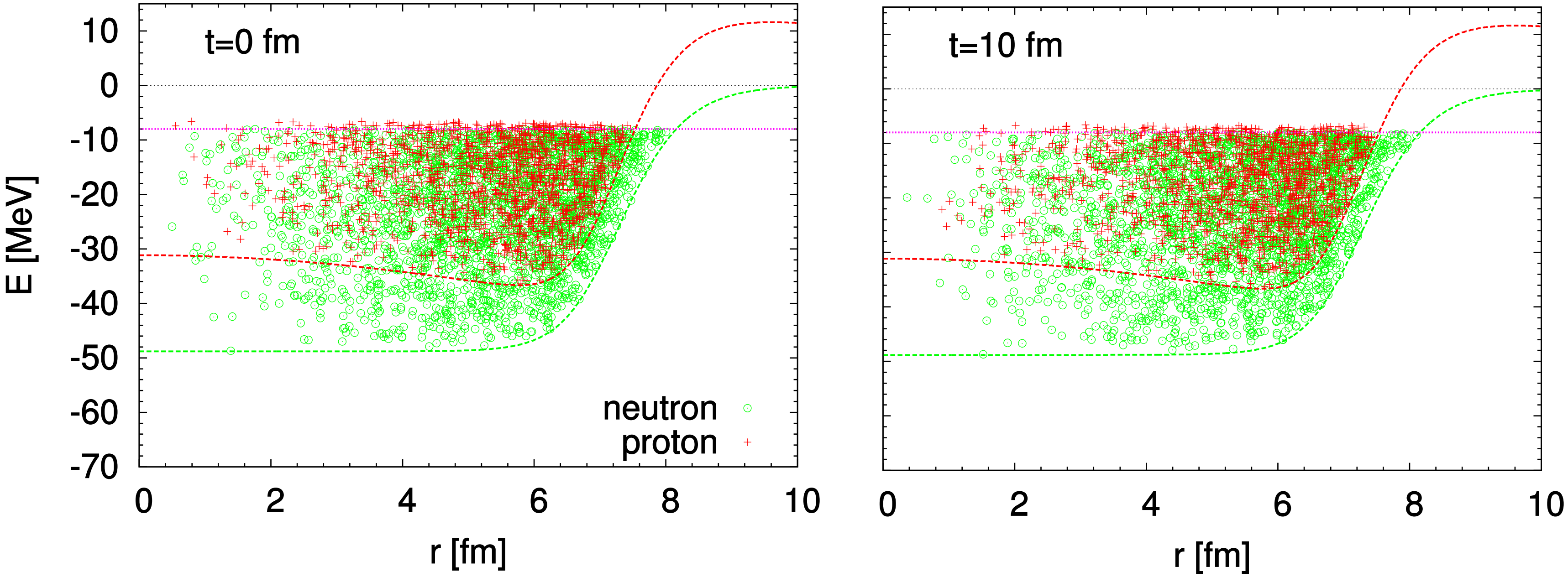
non-mom.dep potential, asymmetry-term, Coulomb



Nuclear ground state

- improvement: ensure constant Fermi-Energy

non-mom.dep potential, asymmetry-term, Coulomb



- needs iteration for mom.dep potential
- important for QE-peak (Gallmeister, Mosel, Weil, PRC94 (2016) 035502)

Init

- in principle:
  1) initialize nucleons
  2) perform **one** initial elementary event on **one** nucleon
  3) propagate nucleons and final state particles
- correct, but 'waste of time'
- *idea:*
  final state particles do not really disturb the nucleus
- 2 particle classes:
  - 'real particles'
  - 'perturbative particles'

Particle classes

- ‘real particles’
  - nucleons
  - may interact among each other
  - interaction products are again ‘real particles’
- ‘perturbative particles’
  - final state particles of initial event
  - may only interact with ‘real particles’
  - interaction products are again ‘perturbative particles’
- ‘real particles’ behave as if other particles are not there
- total energy, total baryon number, etc. not conserved!

Init with perturbative particles

- init
  1) initialize nucleons
  2) perform **one** initial elementary event on **every** nucleon
  3) propagate nucleons and final state particles

  - final states particles are ‘perturbative particles’
  - different final states do not interfere

- every final state particle gets a ‘perturbative weight’:
  - value: cross section of initial event
  - is inherited in every FSI
  - *for final spectra the ‘perturbative weights’ have to be added, not only the particle numbers*

Init with perturbative particles

- *idea:*

  simple workaround against oscillating ground states:
  freeze nucleon testparticles

- since nucleons are real particles, their interactions among each other should not influence final state particles

- advantage: computational time
- disadvantage: ???