

GiBUU

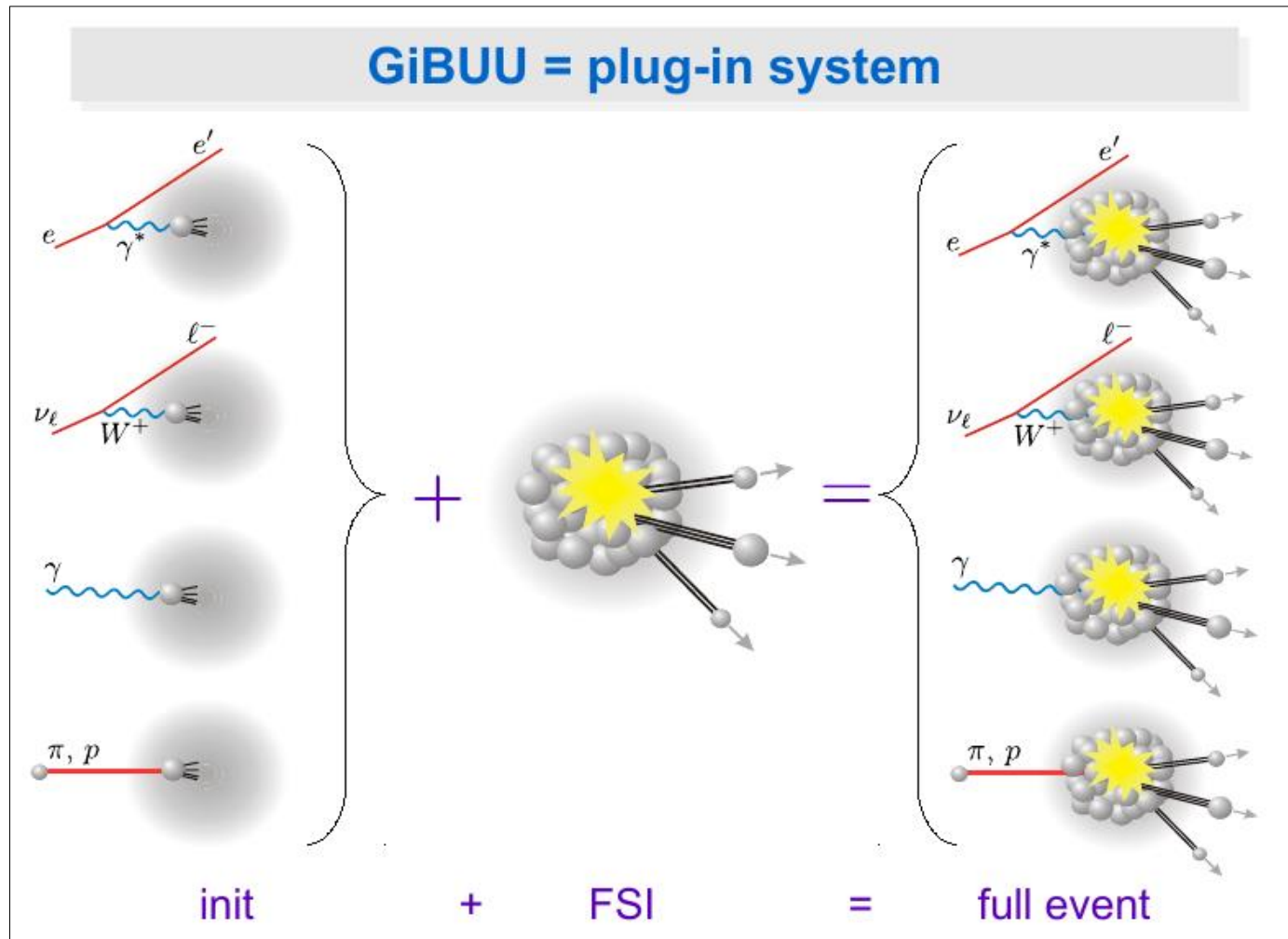
Theoretical Basis

Giessen Boltzmann-Uehling-Uhlenbeck

Kai Gallmeister, Frankfurt

Ulrich Mosel, Giessen

GiBUU in one picture



No factorization because final state of init is initial state of FSI

■ GiBUU

= The Giessen Boltzmann-Uehling-Uhlenbeck Project

■ Theory and Code for simulation of nuclear reactions

■ A+A (~ 1990)

■ hadron+A (p+A, π +A) (~ 1995)

■ γ +A (~ 1998)

■ e+A (~ 2000)

■ ν +A ($\sim 2005 -$)

■ energies: 10 MeV/A \rightarrow 10-100 GeV/A

■ degrees of freedom: Hadrons (Baryons, Mesons)

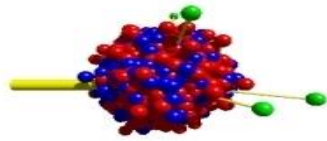
■ propagation and collisions of particles in mean fields

■ approx. Kadanoff-Baym and Boltzmann-Uehling-Uhlenbeck equations solved

GiBUU: Neutrinos

Essential Properties:

1. Theory is semiclassical, i.e. it forgets about quantum coherence and replaces wavefunctions by local plan waves
2. Consequence:
 1. Semi-inclusive reactions such as $(e, e'pX) A$ can be described, but not exclusive $(e, e'p) A$
 2. There are no shell-effects anywhere, only 'average' nuclear properties \rightarrow energy transfers $\sim > 50$ MeV
 3. FSI do not remember the 'interactions before', except for kinematics



- **GiBUU : Quantum-Kinetic Theory and Event Generator**
based on a BM solution of Kadanoff-Baym equations
- GiBUU propagates phase-space distributions, not particles
- Physics content and details of implementation in:
Buss et al, Phys. Rept. 512 (2012) 1- 124
- Code from gibuu.hepforge.org, latest version GiBUU 2019
Details in **Gallmeister et al, Phys.Rev. C94 (2016) no.3, 035502**
- A recent review of generators in general can be found in:
U. Mosel, J. Phys. G, <https://doi.org/10.1088/1361-6471/ab3830>

Quantum-kinetic Transport Theory from non-equilibrium Green's function method

- Kadanoff-Baym (1962) start from eq. of motion for 1-particle Green's function which depends on all n-particle Green's functions
 - Approximations:
 - Truncate hierarchy of coupled Green's functions, i.e. neglect all explicit many-body Green's functions, absorb their effect into modeled self-energies.
 - Gradient approximation: assume that densities vary slowly (good for heavier nuclei, in practice $A \gtrsim 12$).

Quantum-kinetic Transport Theory from non-equilibrium Green's function method

- One-particle density matrices depend smoothly on cm of coordinates, but may oscillate as function of difference
- → Introduce Wigner-transforms, i.e. perform Fourier analysis of difference of space-time coordinates in Green's function

$$G_{\alpha\beta}^<(x, p) = \int d^4\xi e^{ip_\mu\xi^\mu} (i) \langle \bar{\psi}_\beta(x + \xi/2)\psi_\alpha(x - \xi/2) \rangle$$

- G is the Fourier-Transform of the (scalar) one-body density matrix!

Quantum-kinetic Transport Theory from non-equilibrium Green's function method

Trace over Dirac indices (i.e. spin averaging) gives a vector current density

$$F_V^\mu(x, p) = -i \operatorname{tr} (G^<(x, p) \gamma^\mu)$$

$$\begin{aligned} \partial_\mu F_V^\mu(x, p) - \operatorname{tr} [\Re \Sigma^{\text{ret}}(x, p), -i G^<(x, p)]_{\text{PB}} \\ + \operatorname{tr} [\Re G^{\text{ret}}(x, p), -i \Sigma^<(x, p)]_{\text{PB}} = C(x, p) . \end{aligned}$$

with

$$C(x, p) = \operatorname{tr} [\Sigma^<(x, p) G^>(x, p) - \Sigma^>(x, p) G^<(s, p)]$$

$$F_V^\mu = (p^{*\mu} / E^*) F,$$

In a homogeneous medium one gets the KB equation:

$$\mathcal{D}F(x, p) + \operatorname{tr} [\Re G^{\text{ret}}(x, p), -i \Sigma^<(x, p)]_{\text{PB}} = C(x, p)$$

Quantum-kinetic Transport Theory from non-equilibrium Green's function method

$$iG^<(x, p) = +2f(x, p) \Im G^{\text{ret}}(x, p)$$

$$iG^>(x, p) = -2(1 - f(x, p)) \Im G^{\text{ret}}(x, p)$$

This allows to introduce the Spectral Function
 $A(x, p) = \text{imaginary part of } sp \text{ propagator}$

$$F(x, p) = 2\pi g f(x, p) A(x, p)$$

$$\mathcal{D}F(x, p) + \text{tr} [\Re G^{\text{ret}}(x, p), -i\Sigma^<(x, p)]_{\text{PB}} = C(x, p)$$

↓
 Botermans-Malfliet approx

$$\mathcal{D}F(x, p) - \text{tr} \left\{ \Gamma(x, p) f(x, p), \Re G^{\text{ret}}(x, p) \right\}_{\text{PB}} = C(x, p)$$

↙
 Width of spectral function

Quantum-kinetic Transport Theory from non-equilibrium Green's function method

On-shell drift term

Off-shell transport term

Collision term

$$\mathcal{D}F(x, p) - \text{tr} \left\{ \Gamma f, \text{Re}S^{\text{ret}}(x, p) \right\}_{\text{PB}} = C(x, p) .$$

$$\mathcal{D}F(x, p) = \{p_0 - H, F\}_{\text{PB}} = \frac{\partial(p_0 - H)}{\partial x} \frac{\partial F}{\partial p} - \frac{\partial(p_0 - H)}{\partial p} \frac{\partial F}{\partial x} \quad H \text{ contains mean-field potentials}$$

Describes time-evolution of $F(x, p)$

$$F(x, p) = 2\pi g f(x, p) \mathcal{P}(x, p) \leftarrow \text{Spectral function}$$

Phase space distribution

KB equations with BM offshell term
Essential for any in-medium physics

One such equation for each kind of particle: neutrino, nucleon, resonance, meson,
All coupled through mean field potential and collision term C

Collision term

- contains one-, two-, and three-body collisions

$$C = C_{1 \rightarrow X} + C_{2 \rightarrow X} + C_{3 \rightarrow X}$$

(1) resonance decays

(2) two-body collisions

- elastic and inelastic
- any number of particles in final state
- baryon-meson, baryon-baryon, meson-meson

(3) three-body collisions (relevant for pi absorption)

- low energies: cross sections based on resonances



- high energies: string fragmentation

Quantum-kinetic Transport Theory from non-equilibrium Green's function method

Approximations to get to ,Standard Generators: GENIE, NEUT, ..':

1. On-shell (quasiparticle) approximation

$$F(x, p) = 2\pi g \delta[p_0 - E(x, \mathbf{p})] f(x, \mathbf{p})$$

$$\begin{aligned} \left[\partial_t + (\nabla_{\mathbf{p}} E_{\mathbf{p}}) \cdot \nabla_{\mathbf{x}} - (\nabla_{\mathbf{x}} E_{\mathbf{p}}) \cdot \nabla_{\mathbf{p}} \right] f(x, \mathbf{p}) &= \frac{g}{2} \int \frac{d^3 \mathbf{p}_2 d^3 \mathbf{p}'_1 d^3 \mathbf{p}'_2}{(2\pi)^9} \frac{m_{\mathbf{p}}^* m_{\mathbf{p}_2}^* m_{\mathbf{p}'_1}^* m_{\mathbf{p}'_2}^*}{E_{\mathbf{p}}^* E_{\mathbf{p}_2}^* E_{\mathbf{p}'_1}^* E_{\mathbf{p}'_2}^*} \\ &\times (2\pi)^4 \delta^{(3)}(\mathbf{p} + \mathbf{p}_2 - \mathbf{p}'_1 - \mathbf{p}'_2) \delta(E_{\mathbf{p}} + E_{\mathbf{p}_2} - E_{\mathbf{p}'_1} - E_{\mathbf{p}'_2}) \times |\mathfrak{M}_{p p_2 \rightarrow p'_1 p'_2}|^2 \\ &\times [\underbrace{f(x, \mathbf{p}'_1) f(x, \mathbf{p}'_2) \bar{f}(x, \mathbf{p}) \bar{f}(x, \mathbf{p}_2)}_{\text{gain term}} - \underbrace{f(x, \mathbf{p}) f(x, \mathbf{p}_2) \bar{f}(x, \mathbf{p}'_1) \bar{f}(x, \mathbf{p}'_2)}_{\text{loss term}}] \end{aligned}$$

Comparison with Liouville equation identifies f as phase-space density!

→ $F \sim f A$ is *spectral phase space density*, has info on x, p and on off-shellness in A

Quantum-kinetic Transport Theory from non-equilibrium Green's function method

More approximations:

1. Drop all potentials

$$\left(\partial_t + \frac{\mathbf{p}}{M} \cdot \nabla_{\mathbf{x}} \right) f(x, \mathbf{p}) = C(x, \mathbf{p})$$

2. One-particle ansatz for $f(x, \mathbf{p}) = \delta(x - x_i(t))\delta(\mathbf{p} - \mathbf{p}_i(t))$

→ free particle motion on lhs,

on rhs (collision term) obtain mean free path, if density is frozen

→ describe free motion with collisions after mean free path.

Summary:

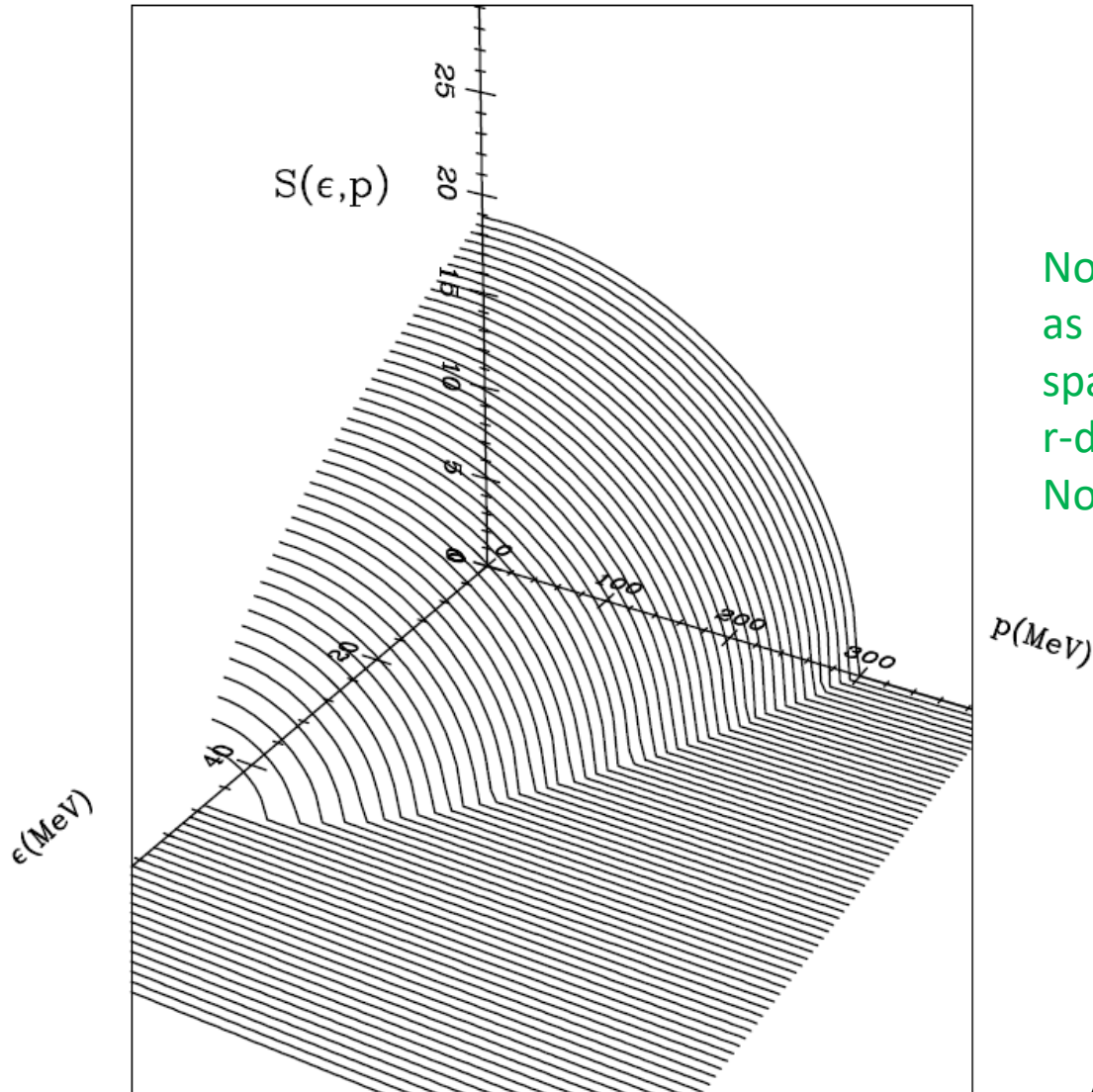
On-shell transport, no potentials, mean free path
approximation to fsi → often used generators

GiBUU Ground State

- Start with empirical density distribution, use reasonable energy-density functional for nuclear matter → calculate mean field potential, dependent on r and p
- Use density to obtain momentum distribution by local Thomas-Fermi model: $k_F^3(r) \sim \rho(r)$
- Readjust k_F slightly to maintain constant Fermi energy

- Spectral Function:
$$\mathcal{P}_h(\mathbf{p}, E) = \int_{\text{nucleus}} d^3x F(\mathbf{x}, t = 0, \mathbf{p}, E)$$
$$= g \int_{\text{nucleus}} d^3x \Theta [p_F(\mathbf{x}) - |\mathbf{p}|] \Theta(E) \delta \left(E - m + \sqrt{\mathbf{p}^2 + m^{*2}(\mathbf{x}, \mathbf{p})} \right)$$

Semiclassical Spectral Function



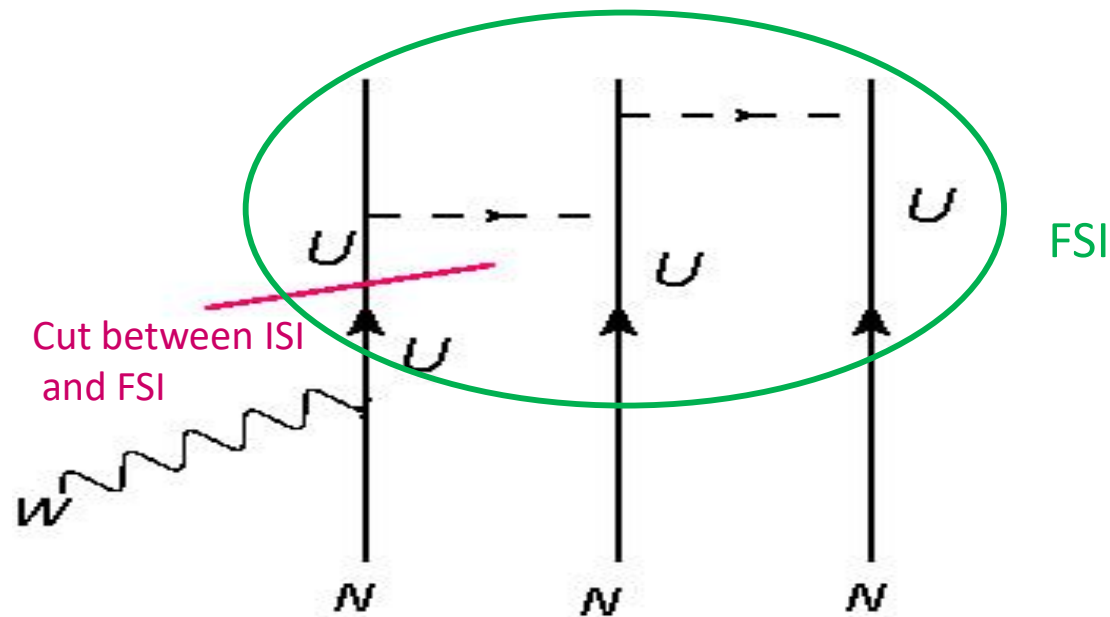
No spiky behavior
as in RFG because of
spatial integration over
 r -dependent potential,
No shell effects

Cross sections on the nucleus

- General procedure used for **all single-particle cross sections** (QE, pion production through resonances, DIS):
 - Calculate cross section for individual nucleons in their restframe, then Lorentz-boost into the Lab-system

QE scattering: Factorization???

Initial state and final state interactions do not factorize

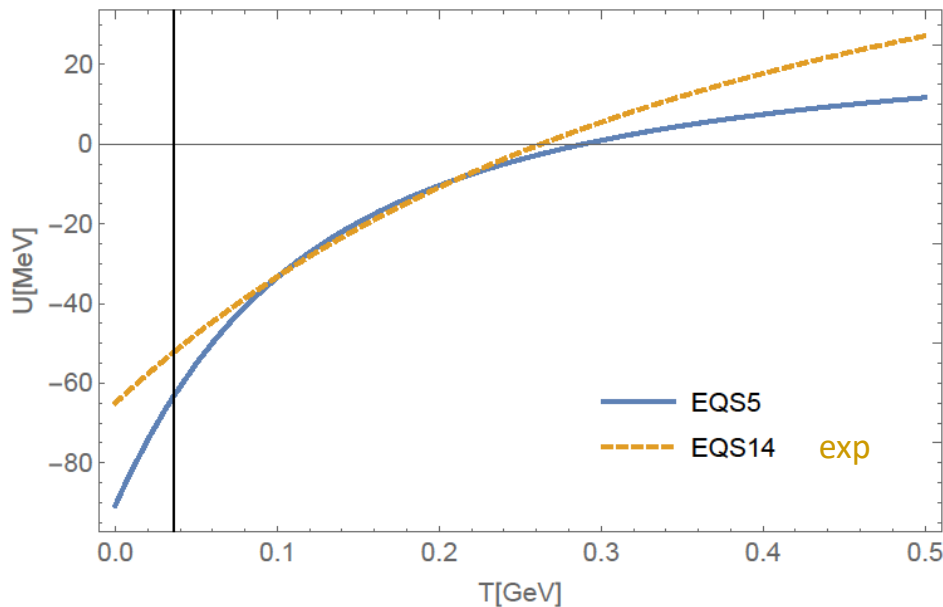


Factorization often invoked by theories, but **not valid**: Same potential U for final state of initial interaction and initial state of final interactions

Potentials in QE scattering

A momentum-dependent potential shifts the position of the QE peak

Experimental pA potential



- Theories without nucleon potentials in the final state work only for reactions with energy transfers $\sim 0.25 - 0.3$ GeV applies to Nieves, Martini, ...GENIE, NEUT, NuWro, SF

2p2h Interactions

Theories for 2p2h interactions:

1. Nieves model, neglects exchange terms, uses local TF as groundstate
2. Martini model, approximates interactions, uses local TF as groundstate
3. Ruiz Simo, Megias model: treatment of Delta–propagator is problematic: no consistent theory for electrons and neutrinos
4. All of these model the MEC with the Delta resonance only, good for MicroBooNE, T2K energies, but not for DUNE
5. All of these give only inclusive cross sections. Need fully exclusive cross sections for energy reconstruction.

GiBUU uses a phenomenological model:

2p2h: Electron induced

■ 2p2h, assume purely transverse

■ electrons

$$\frac{d^2\sigma^{2p2h}}{d\Omega dE'} = \frac{8\alpha^2}{Q^4} E'^2 \cos^2 \frac{\theta}{2} \left(\frac{Q^2}{2\mathbf{q}^2} + \tan^2 \frac{\theta}{2} \right) W_1^e(Q^2, \omega)$$

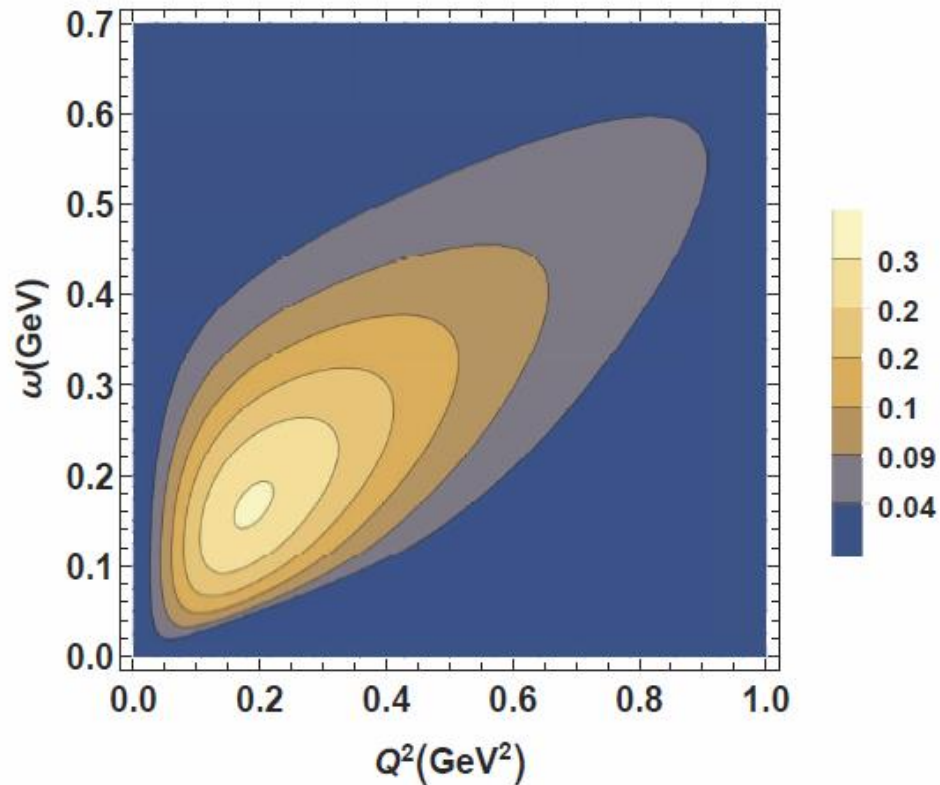
$$W_1^e = G_M^2 \frac{\omega^2}{\mathbf{q}^2} R_T^e \quad R = \text{reduced transverse response}$$

■ W_1 determined from data in a wide kinematical range:

$$0 < W < 3.2 \text{ GeV}, 0.2 < Q^2 < 5 \text{ GeV}^2 \quad (\text{Bosted, Christy})$$

We have no good theory to describe these data → use a phenomenological model, based on this experimental analysis.

Structure Function W_1 (Bosted/Christy)



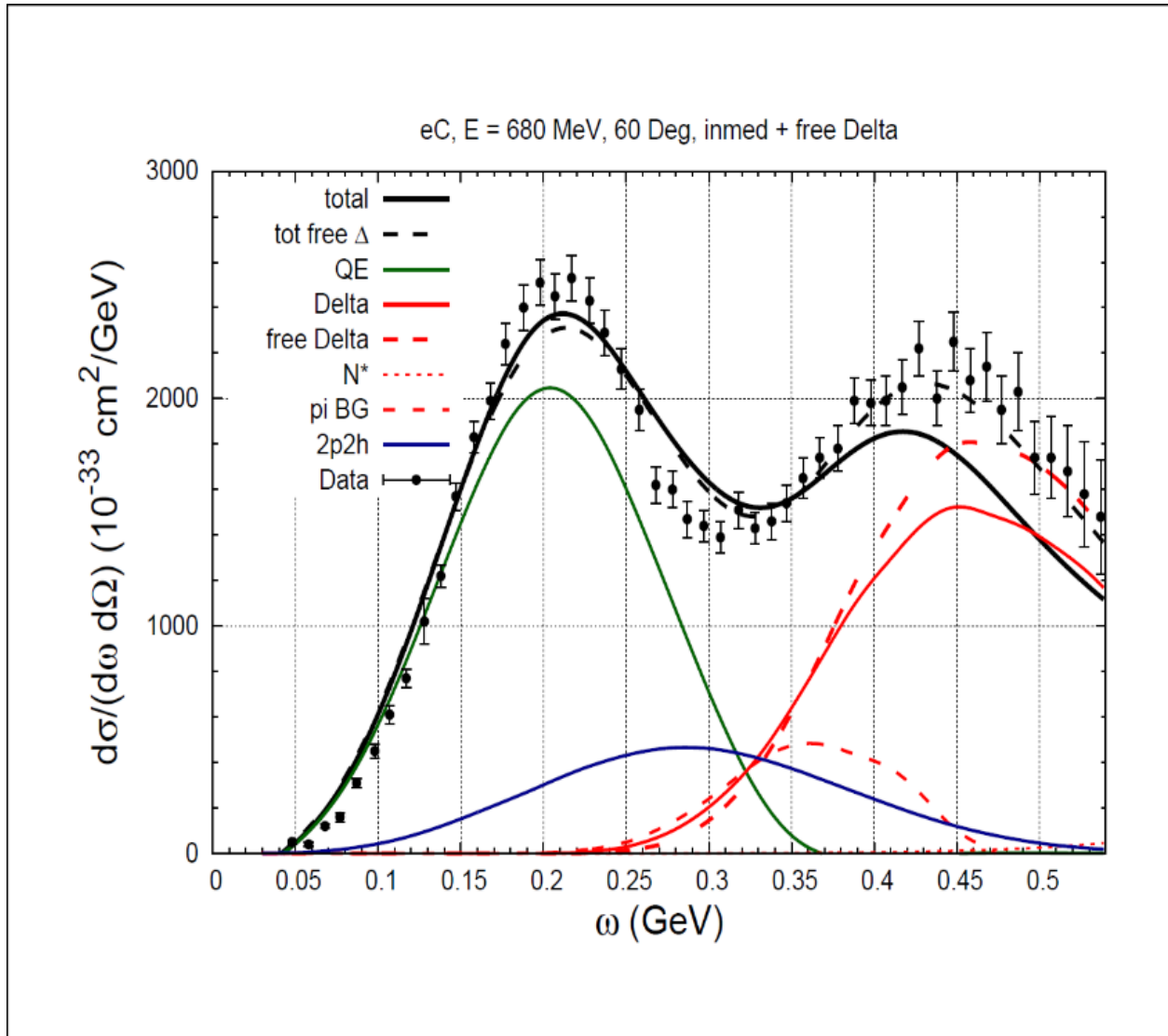
Parametrized in wide kinematical range for **electrons**:

$$0 < W < 3.2 \text{ GeV}, 0.2 < Q^2 < 5 \text{ GeV}^2$$

Problem at $Q^2 = 0$

As an empirical fit it contains implicitly:
src, MEC, nucleon correlations, not just MEC

2p2h: Electron induced



2p2h: Neutrino induced

$$\frac{d^2\sigma^{2p2h}}{d\Omega dE'} = \frac{G^2}{2\pi^2} E'^2 \cos^2 \frac{\theta}{2} \left[2W_1^\nu \left(\frac{Q^2}{2\mathbf{q}^2} + \tan^2 \frac{\theta}{2} \right) \mp W_3^\nu \frac{E + E'}{M} \tan^2 \frac{\theta}{2} \right]$$

Purely transverse, insensitive to gauge transfo problems

Walecka, O'Connell, Donnelly, Walecka (1972); connects electron response with neutrino response: invariance

$$W_1^\nu = \left(G_M^2 \frac{\omega^2}{\mathbf{q}^2} + G_A^2 \right) R_T^e 2(\mathcal{T} + 1)$$

\mathcal{T} = isospin of target nucleus

Now assume:

- longitudinal current negligible in W_3
- Neutrinos populate isobaric analogues of electron-excitations

$$W_3^\nu = 2G_A G_M R_T^e 2(\mathcal{T} + 1)$$

Same expressions used by Ericsson, Delorme (1985), Martini et al (2009 - ..)

One structure function determines electrons, neutrinos and antineutrinos

Propagation of 2p2h events

So far, only inclusive X-sections, but for event generation need the 2 outgoing nucleons
from initial neutrino-2p2h interaction

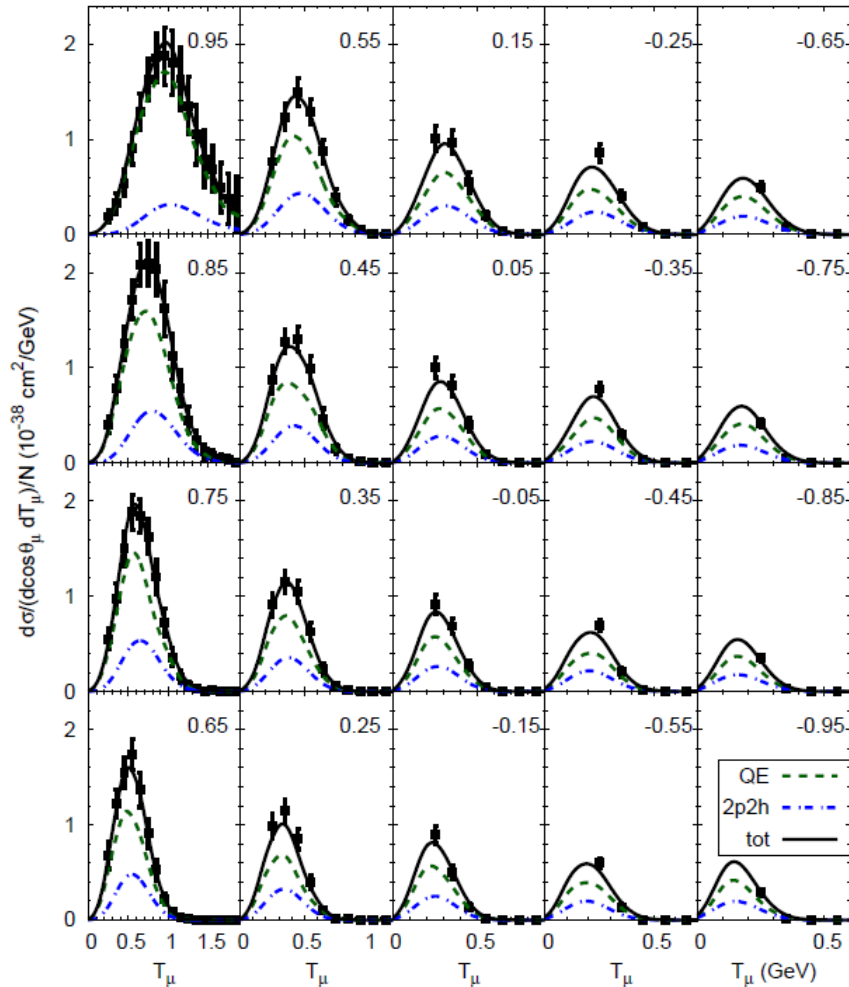
- Choose initial nucleons with random momenta inside the Fermi sea, but at same location (short range assumption)
- Choose isospin according to simple combinatorics (no special weight for pn vs nn ..)
- Go to cm-system of 2 nucleons after absorption of momentum transfer q :

$$P_{\text{cm}} = p_1 + p_2 + q = 0$$

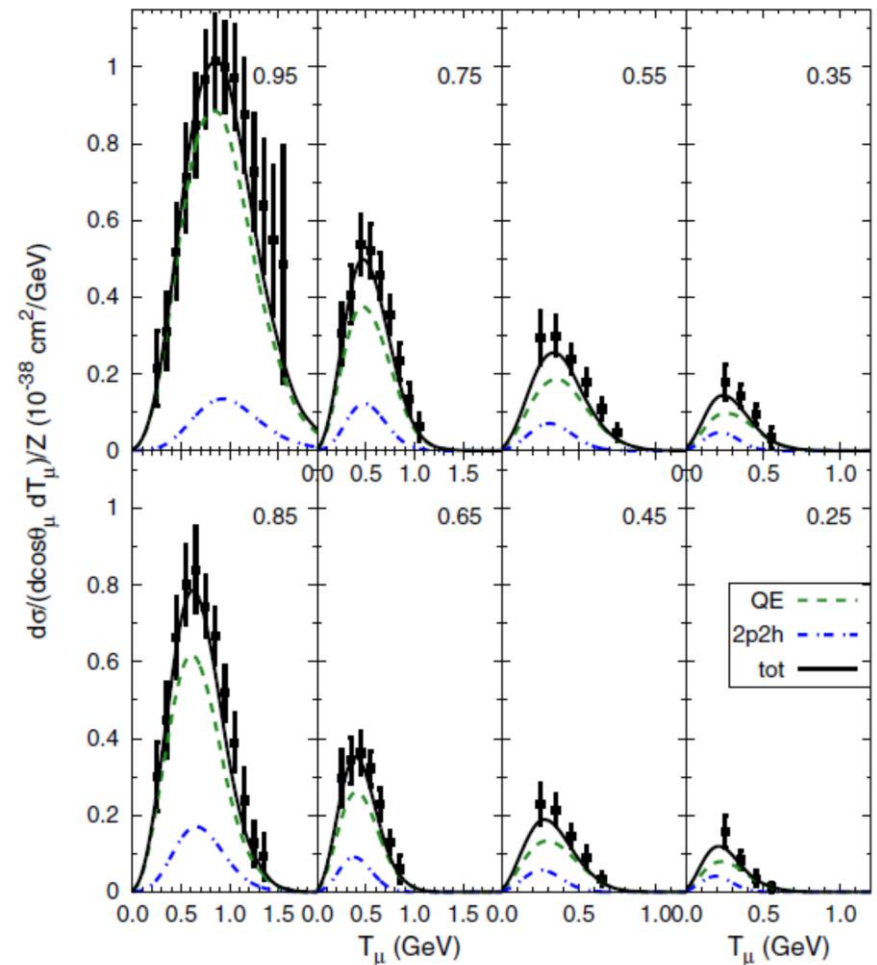
- In that cm system populate final nucleon states according to phase space (somewhat complicated because potential is momentum-dependent!)
- Propagate these two nucleons as usual, with all fsi (potential and collisions)

MiniBooNE 0pion = QE + 2p2h

neutrinos

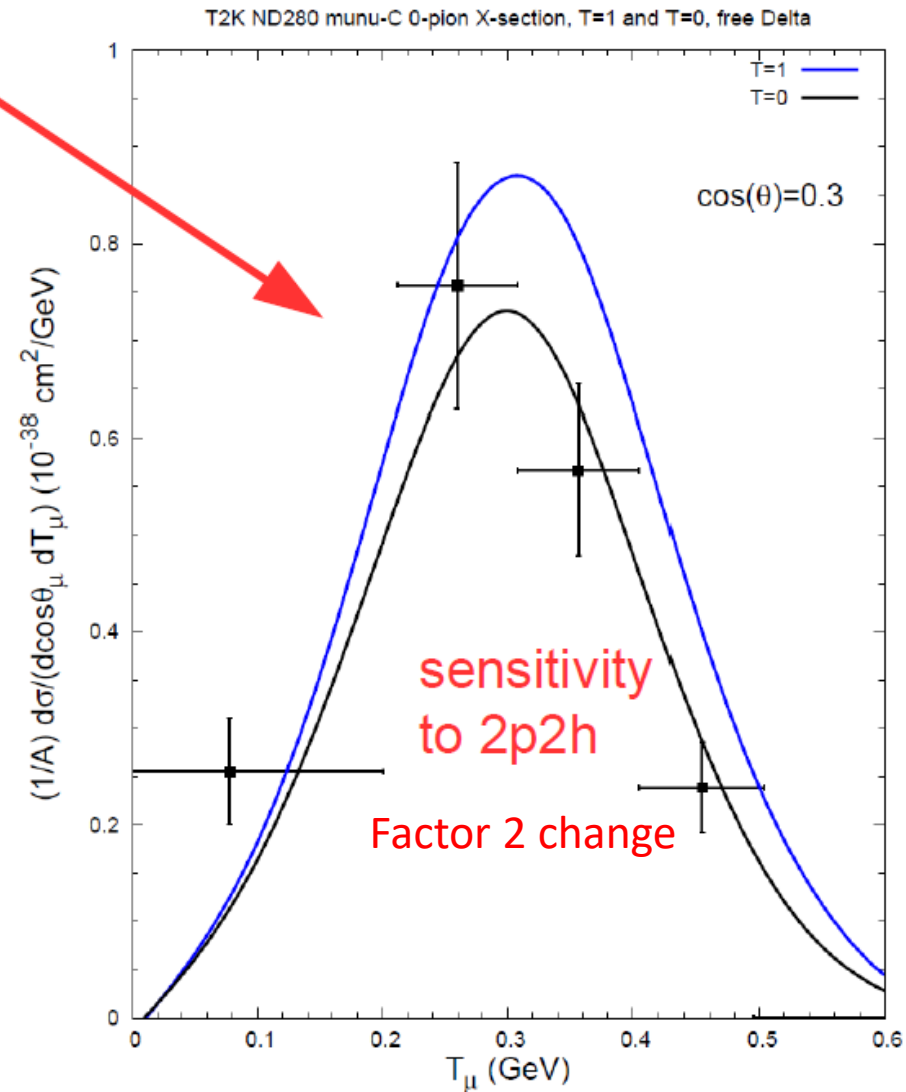
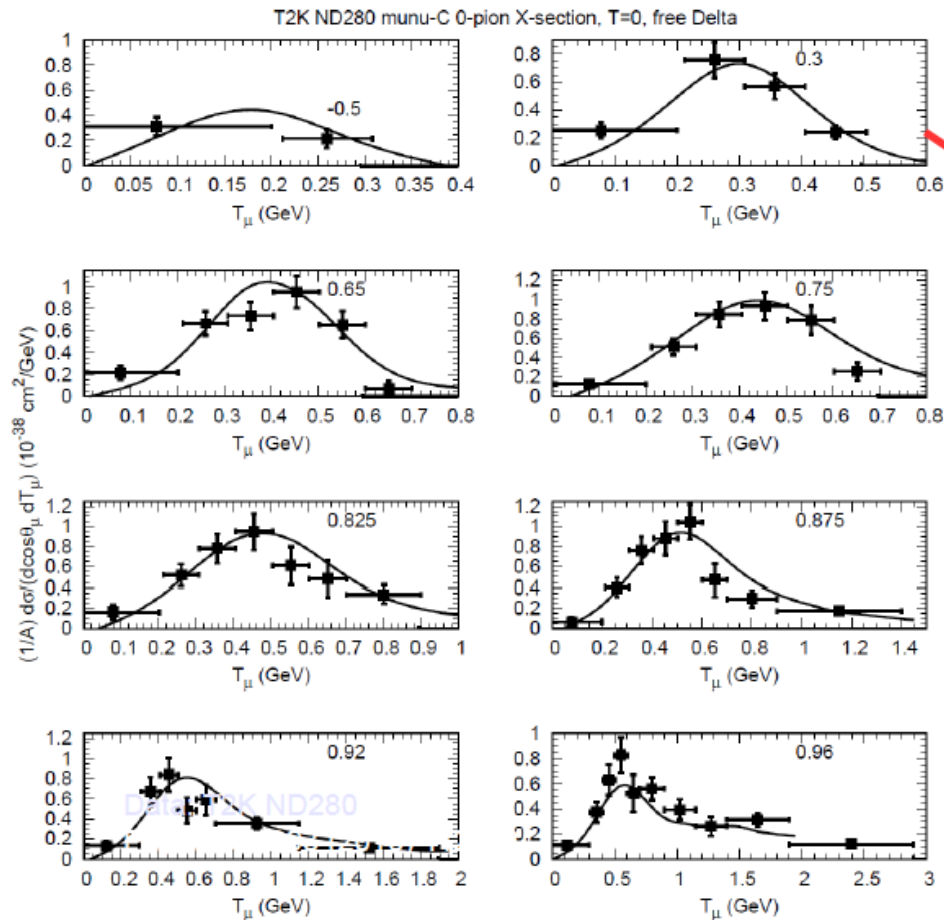


antineutrinos



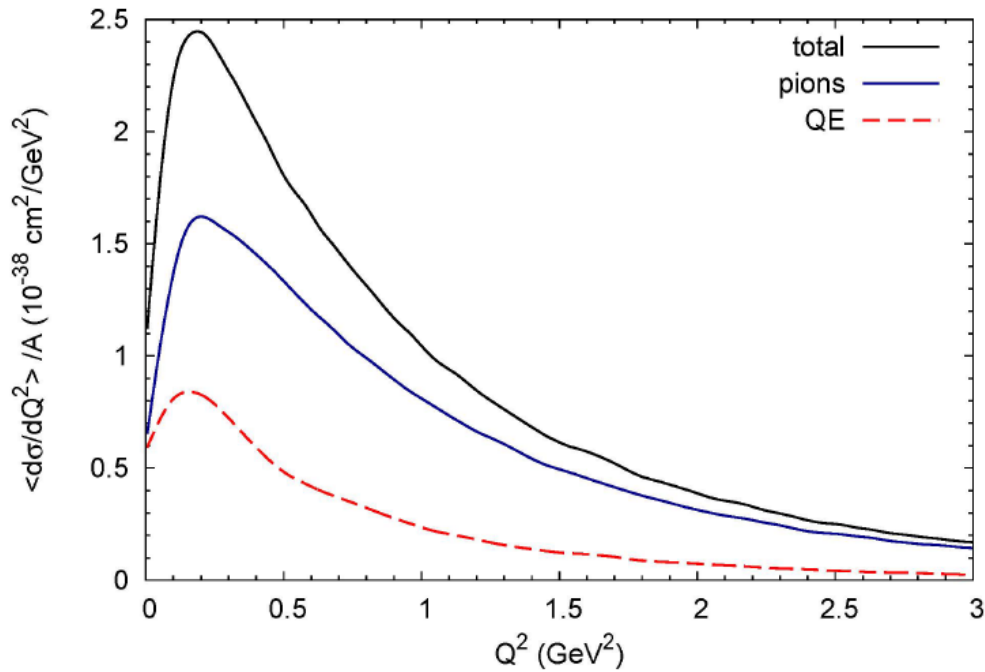
No flux correction!

T2K 0-pion = QE + 2p2h + stuck pions



Data: T2K ND280
Phys.Rev. D93 (2016) 112012

Pion Production



DUNE flux on Ar40

QE= true QE + 2p2h

pions= resonances + DIS

➔ Pions are dominant!

must be under
quantitative control

Elementary Cross section

Pion production in resonance region ($W \sim < 2 \text{ GeV}$) has resonance and background amplitudes

$$\sigma \propto |A_R + A_{BG}|^2 = |A_R|^2 + |A_{BG}|^2 + \textit{interference}$$

We obtain both from MAID2007 analysis for $W < 2 \text{ GeV}$ of electron- and photon-induced pion production on the nucleon
→ Electron cross section on *nucleon* is correct by construction

$$\sigma \propto |A_R|^2 + \textit{BGterms}$$

We propagate the resonances, but not the BG terms;
the BG terms can be < 0 !

Pion production on the nucleon

Transition currents to resonances:

$$V_{3/2}^{\alpha\mu} = \frac{C_3^V}{M} (g^{\alpha\mu} \not{q} - q^\alpha \gamma^\mu) + \frac{C_4^V}{M^2} (g^{\alpha\mu} q \cdot p - q^\alpha p^\mu) + \frac{C_5^V}{M^2} (g^{\alpha\mu} q \cdot p - q^\alpha p^\mu) + g^{\alpha\mu} C_6^V$$

$$A_{3/2}^{\alpha\mu} = - \left[\frac{C_3^A}{M} (g^{\alpha\mu} \not{q} - q^\alpha \gamma^\mu) + \frac{C_4^A}{M^2} (g^{\alpha\mu} q \cdot p - q^\alpha p^\mu) + C_5^A g^{\alpha\mu} + \frac{C_6^A}{M^2} q^\alpha q^\mu \right] \gamma^5 .$$

C^V from electron data (MAID analysis with CVC)

C^A from fit to neutrino data (experiments on hydrogen/deuterium), so far only C_5^A determined, for other axial FFs only educated guesses

Vertex factor $\Gamma^{\alpha\mu} = (V^{\alpha\mu} - A^{\alpha\mu}) \gamma^5$

Hadron tensor $H^{\mu\nu} = \frac{1}{2} \text{Tr} [\not{p} + M) \Gamma^{\alpha\mu} \Lambda_{\alpha\beta} \Gamma^{\beta\nu}]$

Contract lepton tensor with hadron tensor gives the resonance production cross section:

$$\frac{d\sigma^{\text{med}}}{d\omega d\Omega'} = \frac{|\mathbf{k}'|}{32\pi^2} \frac{\mathcal{P}^{\text{med}}(p')}{[(k \cdot p)^2 - m_\ell^2 M^2]^{1/2}} |\mathcal{M}_R|^2$$

Formalism on Nucleon

$$d\sigma(\nu p \rightarrow \ell^- p \pi^+) = \sum_{\substack{I=3/2 \\ \text{resonances}}} b_i d\sigma_{R_i^{++}},$$

$$d\sigma(\nu n \rightarrow \ell^- n \pi^+) = \frac{1}{3} \sum_{\substack{I=3/2 \\ \text{resonances}}} b_i d\sigma_{R_i^+} + \frac{2}{3} \sum_{\substack{I=1/2 \\ \text{resonances}}} b_i d\sigma_{R_i^+},$$

$$d\sigma(\nu n \rightarrow \ell^- p \pi^0) = \frac{2}{3} \sum_{\substack{I=3/2 \\ \text{resonances}}} b_i d\sigma_{R_i^+} + \frac{1}{3} \sum_{\substack{I=1/2 \\ \text{resonances}}} b_i d\sigma_{R_i^+},$$

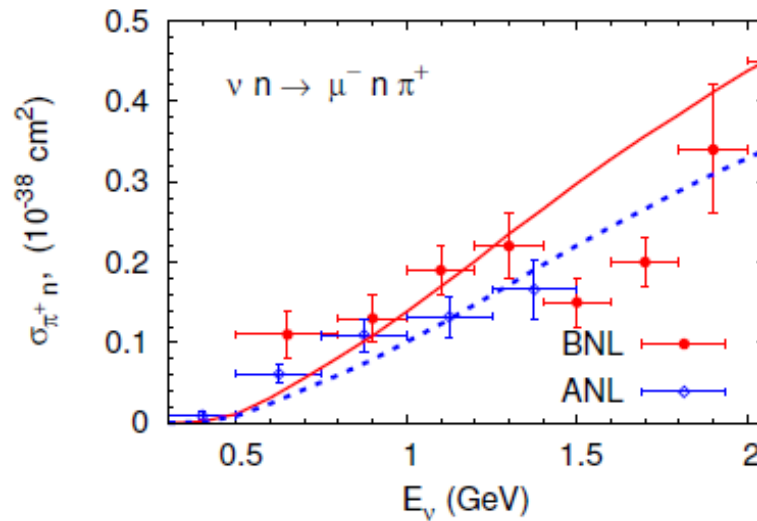
branching ratios $b_i = \Gamma_{\pi N} / \Gamma_{\text{tot}}$

In the vector sector data are described because we use MAID07 analysis
Higher excitations with $W > 2$ are handled by DIS processes through PYTHIA

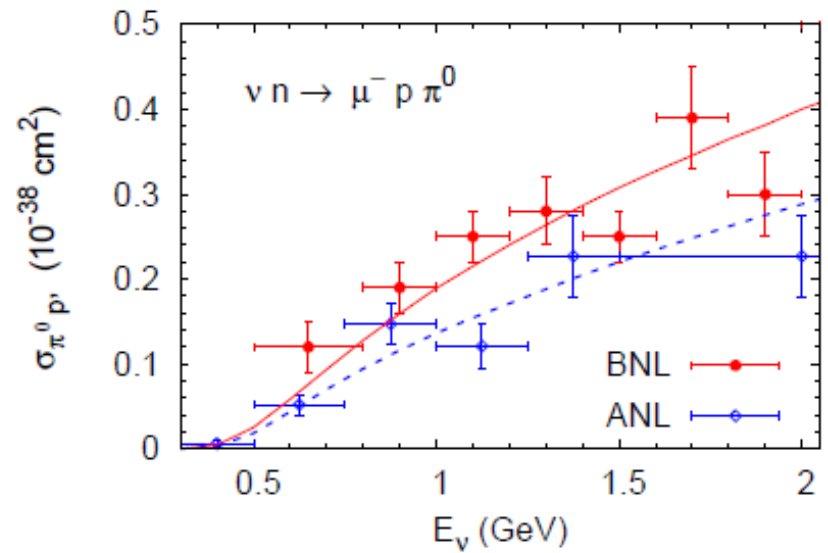
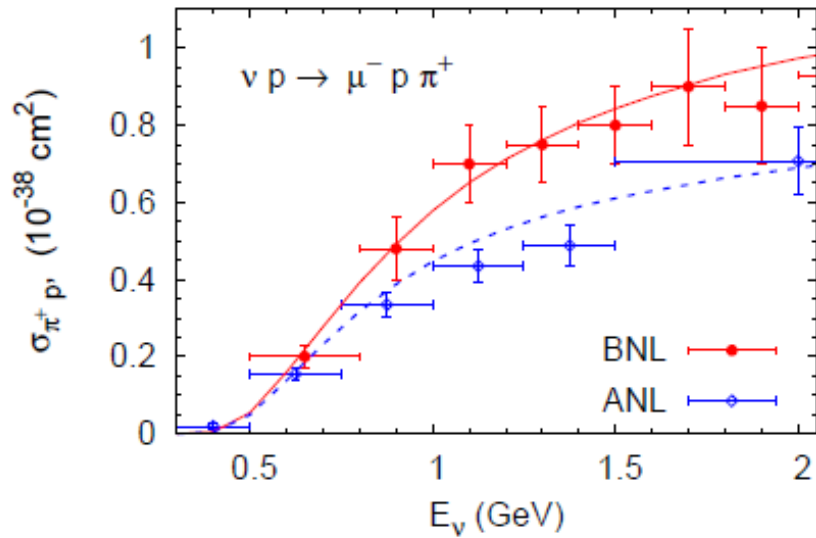
BG parameters:

- for electrons from MAID analysis
- for neutrinos are obtained by fit to nucleon data

Elementary Cross Sections



ANL
is now default



Formalism on Nucleus

Integrate the nucleon cross sections over the Fermi-sea of bound nucleons

$$d\sigma^A = \int \frac{d^3p}{(2\pi)^3} dE P_h(p, E) d\sigma^N P_{PB}$$

Hole spectral function

Pauli blocking

Resonances and nucleons sit in potential,
Delta potential is weaker than nucleon potential ($\sim 2/3$)

Final State Interactions of Pions

- Two-body pi absorption through $\pi + N \rightarrow \Delta, \Delta + N \rightarrow NN$
- Three-body pi absorption:

$$\Gamma_{N_A N_B \pi \rightarrow N_a N_b} = \Gamma_{N_A N_B \pi \rightarrow N_a N_b}^{\text{BG}} + \Gamma_{N_A N_B \pi \rightarrow N_a N_b}^{\text{resonance contribution}}$$

$$\Gamma_{N_A N_B \pi \rightarrow N_a N_b}^{\text{BG}} \sim \sigma_{NN \rightarrow NN \pi}^{\text{BG}}$$

$$\Gamma_{N_A N_B \pi \rightarrow N_a N_b}^{\text{resonance contribution}} \sim \sigma_{NN \rightarrow NN \pi}^{\text{resonance contribution}}$$

Pion Production and Absorption

In resonance (N^*) region time reversal invariance requires



Pion production



Pion absorption

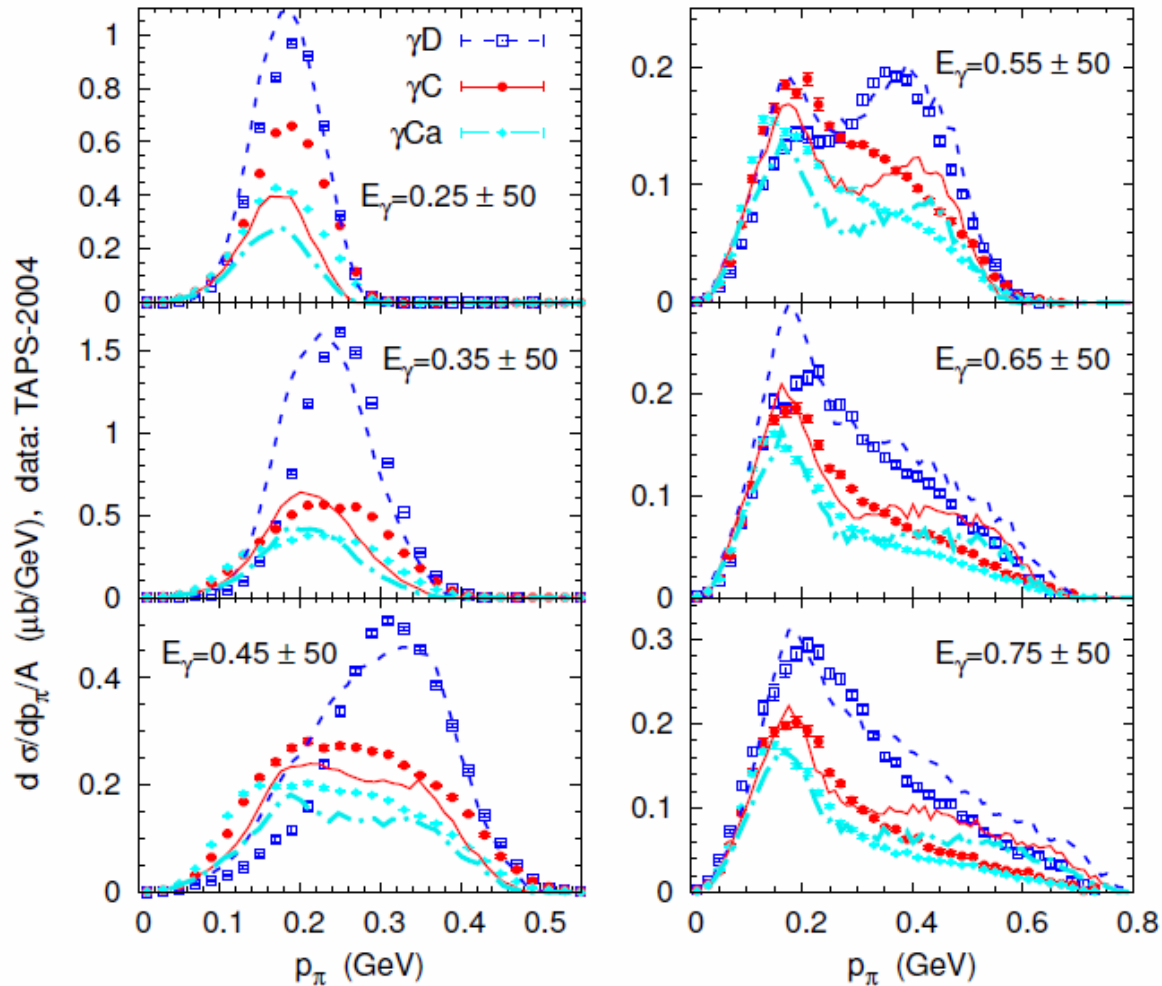


- Generators GENIE, NEUT, NuWro all violate this basic principle:
 - pion production usually taken from Rein-Sehgal (38yrs old!, outdated)
 - Pion absorption taken from Valencia model
 - Not verified with electron data

These generators contain artificial degrees of freedom
and tuning parameters

Test with γA

- $\gamma A \rightarrow \pi^0$ TAPS data



Targets:

D

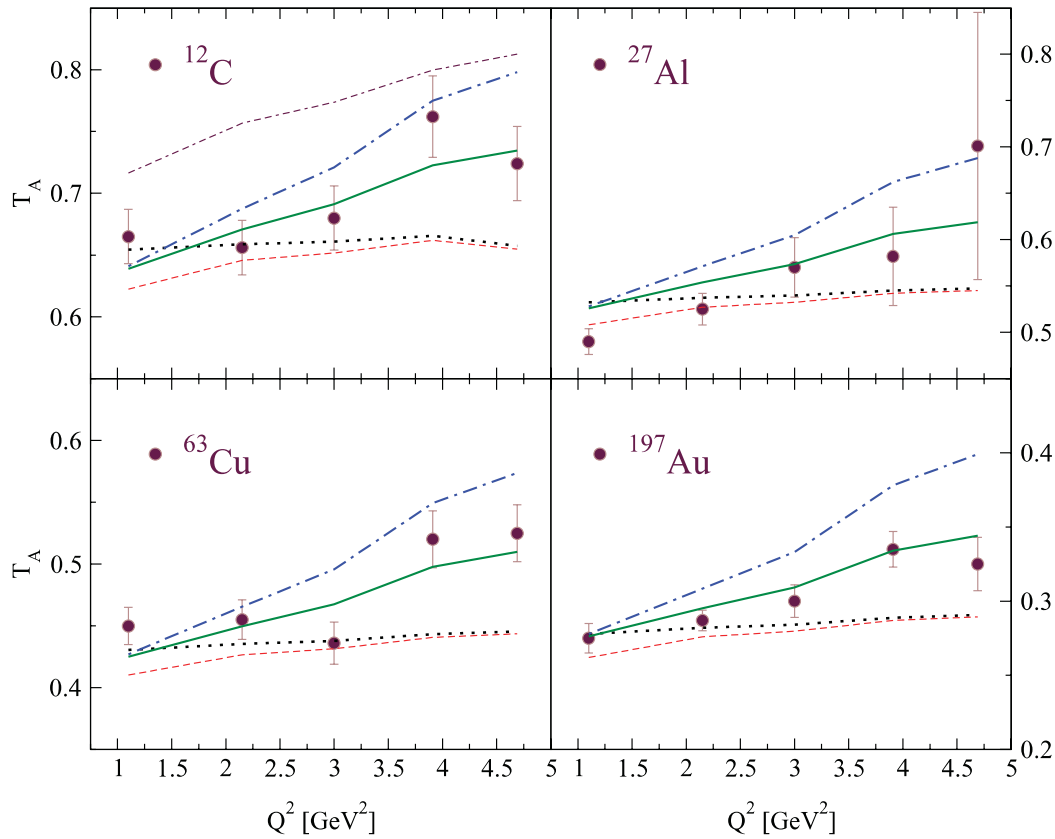
C

Ca

Lalukulich et al,
AIP Conf.Proc.
1663 (2015) 040004

Test with eA

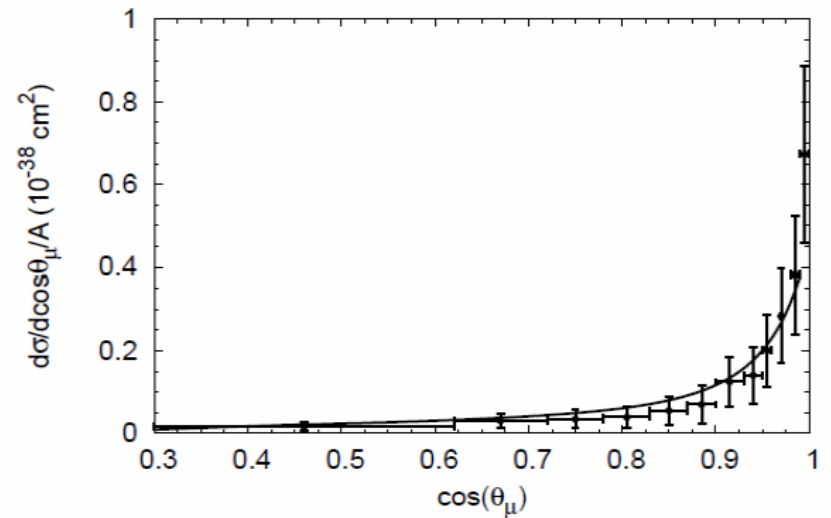
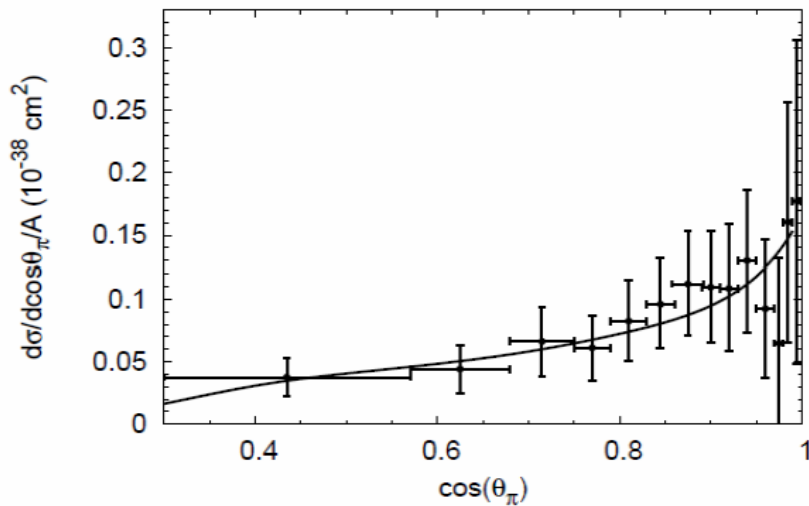
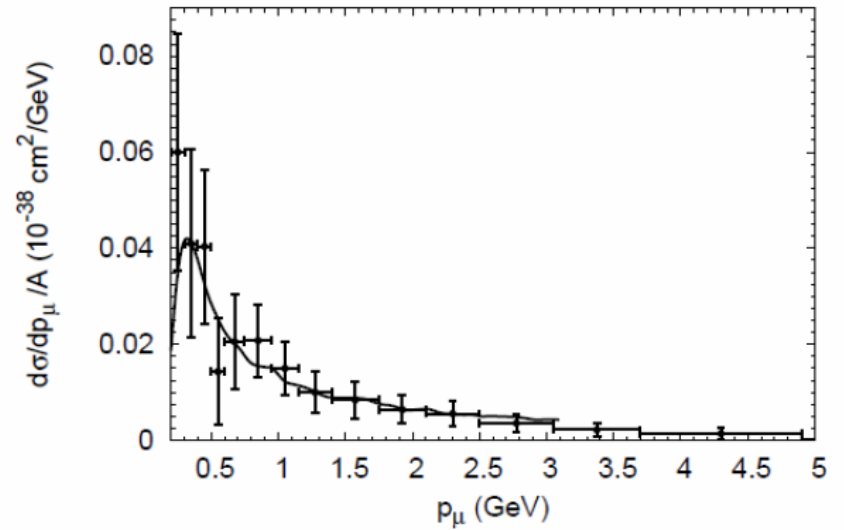
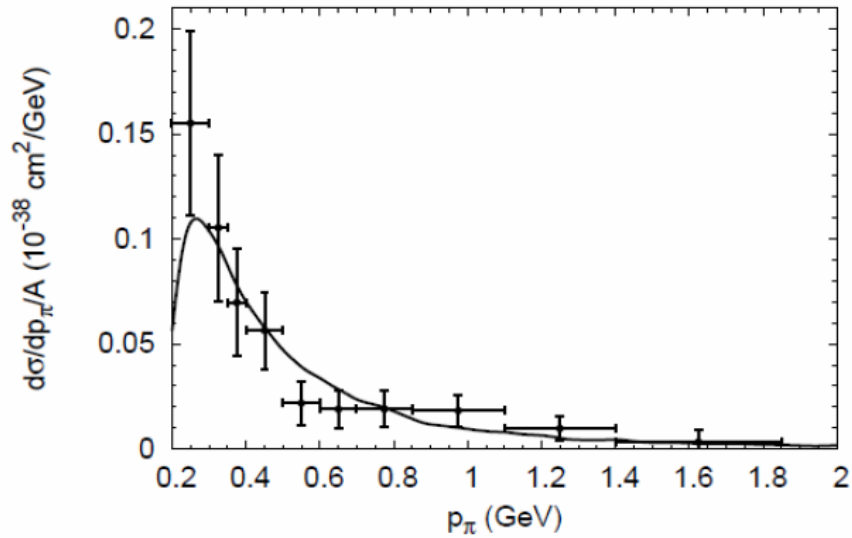
- π^+ JLAB data



Exp: B. Clasie et al.
Phys. Rev. Lett. 99, 242502 (2007).

GiBUU: Kaskulov et al,
Phys.Rev. C79 (2009) 015207

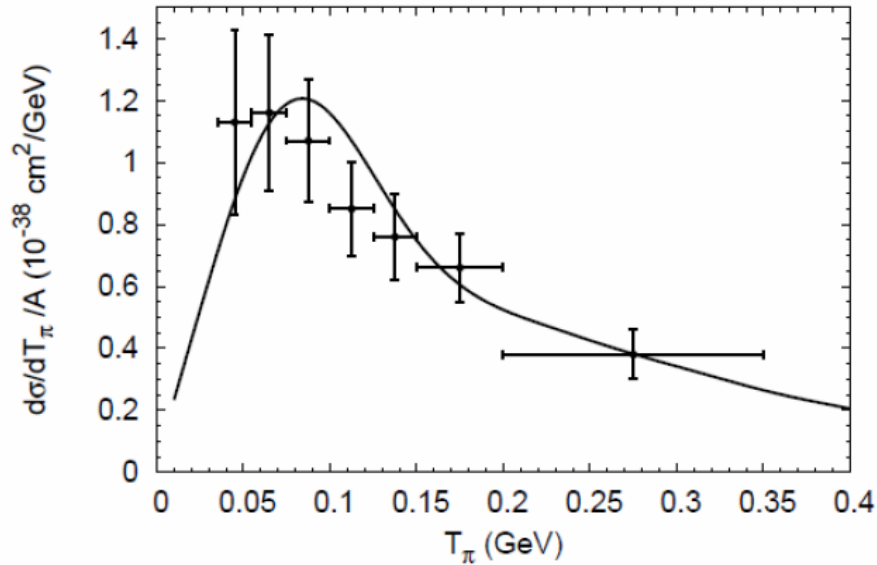
T2K ND280 pions on water



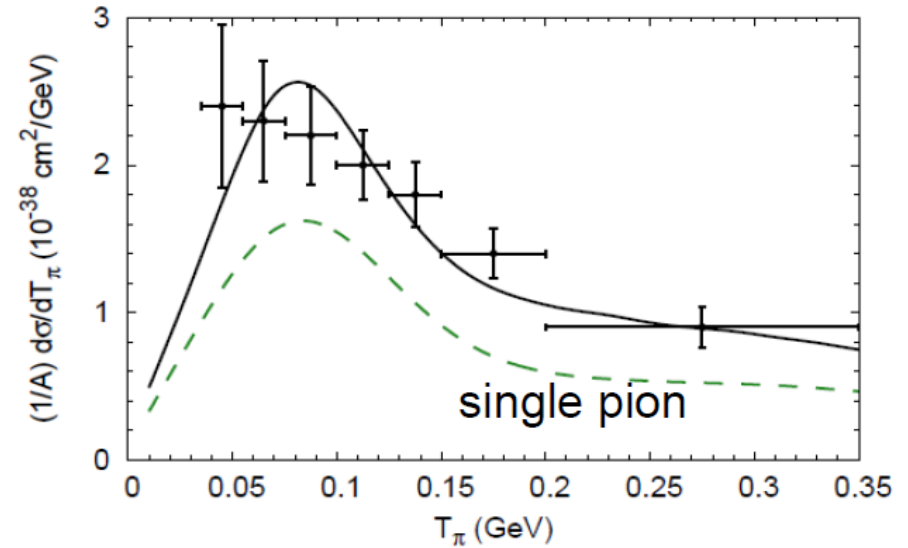
Data: T2K ND280
Phys.Rev. D95 (2017) 012010

MINERvA pions

■ CC charged pions



$W < 1.4 \text{ GeV}$



$W < 1.8 \text{ GeV}$, multiple pions

DIS in GiBUU

1. Calculate total DIS cross section from pQCD, using standard pdfs
2. Obtain mass- and energy-distribution of final state particles from string-fragmentation (PYTHIA)

(Lund) String-fragmentation (Pythia)

■ *idea:*

hard qq scattering (pQCD)
creates a color flux tube ('string')
which then fragments into hadrons
(via qq pair production)

■ high energy: 10 GeV...

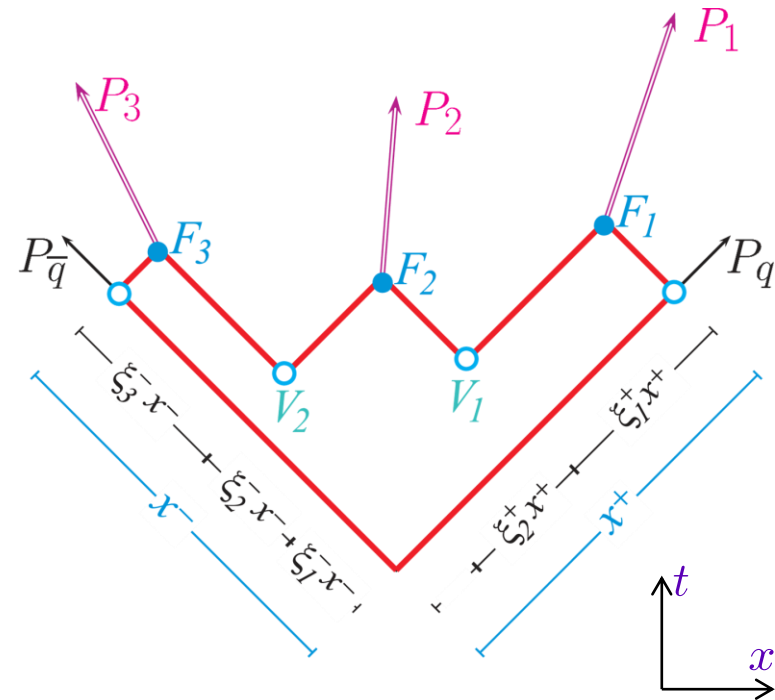
■ "Lund string model"

implementation: Pythia (Jetset)

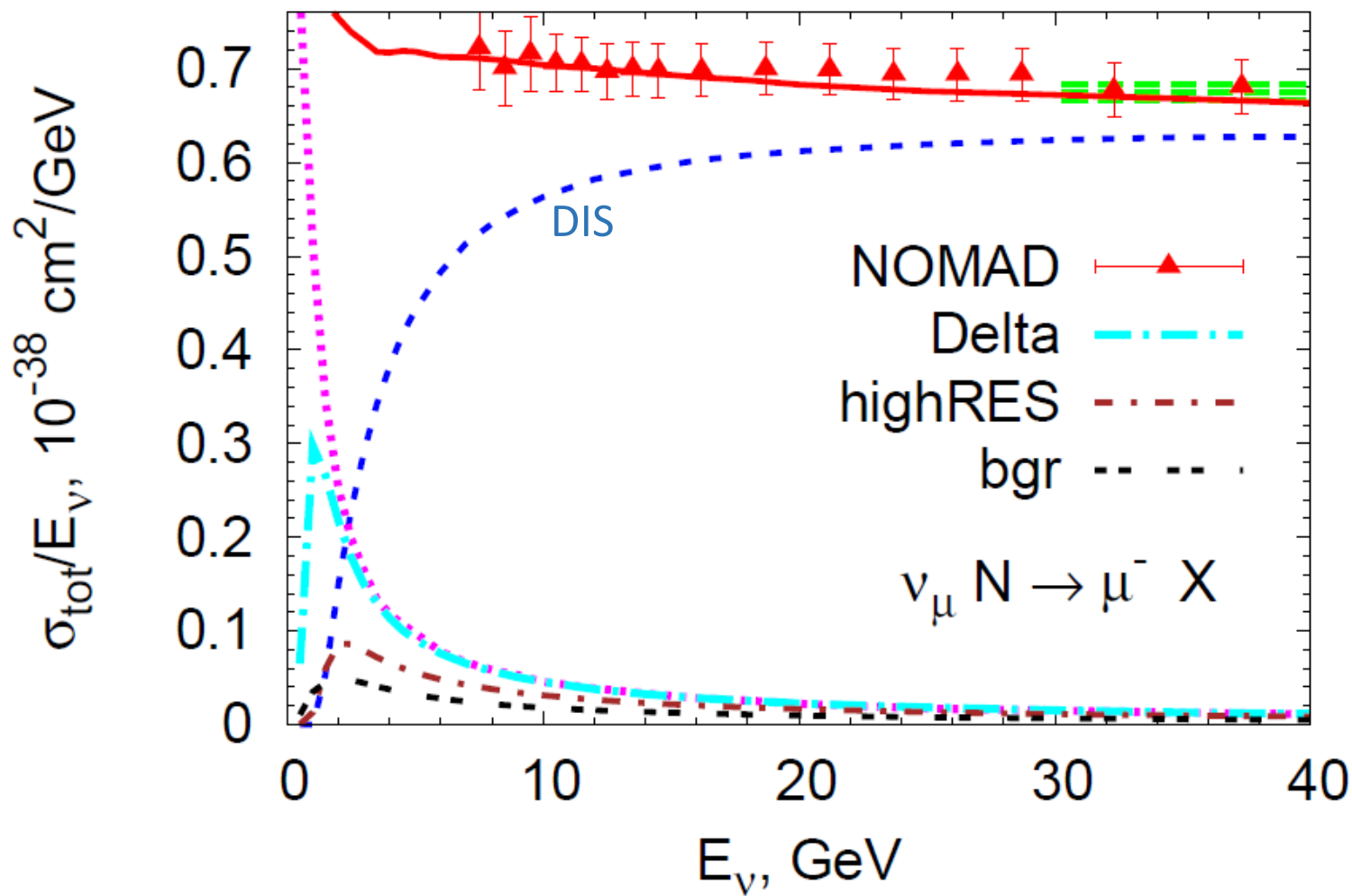
■ only low-lying resonances

■ phenomenological fragmentation function
(when and how does a string break?)

■ parameters fitted to data (different 'tunes', e.g. to HERMES data, available)



DIS in GiBUU



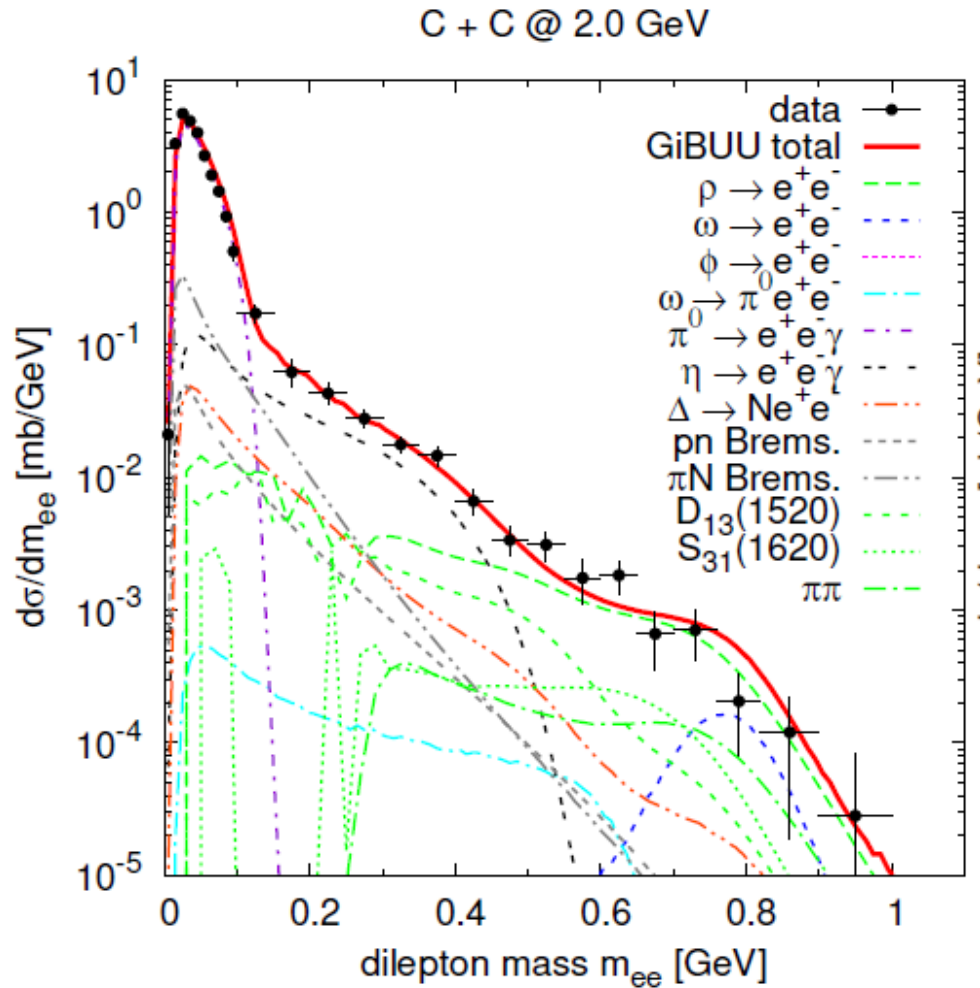
Final State Interactions

- **Neutrinos** ,illuminate' the whole nucleus: final state particles can be produced anywhere in the nuclear volume. Need mean free path at different densities and momenta
- **Generators** (GENIE, ...) replace this by tuning
 $hadron + A$
cross sections \rightarrow **incorrect geometry**, insensitive to pathlengths in medium
- Hadron absorption cross sections, e.g. $\pi + A \rightarrow X$ do not test the needed pion mean free path in nuclei, but just the overall absorption rate
- Ideal test: (γ, A) and (e, A) reactions both illuminate also whole nucleus

Propagation of Hadrons

- Hadrons are propagated within their self-energies, i.e. potentials (nucleons always, mesons optional)
- Because of potentials nucleon trajectories are not straight lines, as in MC, but have to be time-integrated
- → increases computing time, but no need to introduce tunable fudge factors such as a binding energy: nucleons become unbound when they leave the nucleus

Test for inverse Reaction: timelike photon production



Dilepton spectrum in the HADES experiment

Summary

GiBUU's essential properties:

1. Consistency between different reaction channels (QE, pion production, ...): same groundstate, no redundant, unphysical degrees of freedom
2. Bound groundstate, no need for binding energy corrections
3. Produces not only inclusive X-sections (such as Scaling, Spectral Function, GFMC methods), but full event final state files, 4-vectors for all particles
4. Time-evolution of the reaction is based on transport theory, as in QGP generators

GiBUU Practical Points

- The code can be downloaded from gibuu.heforge.org
- The code is documented by robodoc: generates documentation on homepage
- The code generates event files with four-vectors of all outgoing particles. This info can be used to put in detector acceptances,
- The code also produces many semi-inclusive differential cross sections. These are calculated without any cuts etc
- Running time: \sim 1-2 hours for inclusive X-sections without time-development, \sim 1 day for fully exclusive events