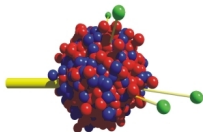


THE GiBUU TRANSPORT TUTORIAL (PART 1)

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FIAS, Frankfurt

HADES Winter School
Kloster Höchst, Feb. 2014



GiBUU

The Giessen Boltzmann-Uehling-Uhlenbeck Project

GENERAL SCOPE AND AIMS OF THE COURSE

- short introduction to general ideas of kinetic theory
- give a basic understanding of how a transport simulation works
- provide a practical introduction to running numerical transport simulations with the GiBUU code
- enable students to run simulations of $p+A$ and $A+A$ reactions in the few-GeV regime
- understand important input parameters of the model
- learn to interpret and analyze the produced output
- cannot cover whole model, but at least provide a baseline for further investigations

Part I (Tue, Feb. 25)

- transport basics
- introduction to the model
- what physics is included
- preparations, getting started
- basic input parameters and output format

Part II (Wed, Feb. 26)

- running simple collisions (p+p)
- analyzing output
- varying input parameters
- time evolution, potentials/EOS, ...
- nuclear reactions (p+A and A+A)
- advanced topics

MODEL INTRO: WHAT GiBUU CAN DO

- GiBUU: The Giessen BUU model (traditionally developed at University of Giessen, now: most contributors scattered around Giessen and Frankfurt)
- flexible tool for numerical simulations of nuclear reactions
- can handle various types of reactions:
 - elementary: pp , pn ('input')
 - electroweak: γA , eA , νA
 - hadronic: pA , πA
 - heavy-ion: AA
- energies: \sim tens of MeV – tens of GeV
- based on the Boltzmann-Uehling-Uhlenbeck (BUU) equation: propagation and collisions of particles in a mean field
- degrees of freedom: hadrons (various flavors of baryons and mesons)
- similar models: UrQMD, HSD, IQMD, BRoBUU, ...

- GiBUU = The **Gi**essen **BUU** model
- 'official' pronunciation: ghee - bee - you - you
- popular alternatives:
 - gee - bee - you - you (as in "Bee Gees")
 - giii - buuh (a la "Hui-Buh")



pick whatever suits your tongue ;)

BASICS: SOME KINETIC THEORY

- distr. function $f(x, p)$ with $x = (t, \vec{x})$, $p = (E, \vec{p})$
- describes (density) distribution of particles in phase space
- number of particles in a given phase-space volume:
$$\Delta N = f(x, p) \Delta^3 x \Delta^3 p$$
- we can set up one distribution function for each particle species: $f_N, f_\pi, f_\Delta, \dots$
- continuity equation for free noninteracting particles:

$$p^\mu \partial_\mu f(x, p) = 0$$

(free straight-line propagation of particles, no collisions)

- adding external force (mean-field potential): Vlasov equ.

$$[\partial_t + (\nabla_p E) \nabla_r - (\nabla_r E) \nabla_p] f(x, p) = 0$$

(propagation through mean field without collisions)

- forget about mean fields, but add collisions ...
- continuity equ. + collision term \Rightarrow Boltzmann eq.

$$p^\mu \partial_\mu f(x, p) = C(x, p)$$

- collision integral C has gain and loss term

$$C(x, p) = C_{gain}(x, p) + C_{loss}(x, p)$$

- including mean field and collision term yields the Boltzmann-Uehling-Uhlenbeck (BUU or VUU) equation

$$(\partial_t + (\nabla_{\vec{p}} H_i) \nabla_{\vec{r}} - (\nabla_{\vec{r}} H_i) \nabla_{\vec{p}}) f_i(\vec{r}, t, \vec{p}) = C[f_i, f_j, \dots]$$

- collision term also includes Pauli blocking (quantum effect)

THE BOLTZMANN-UHRLING-UHLENBECK EQUATION

- BUU equ. describes space-time evolution of phase-space density $f_i(\vec{r}, t, \vec{p})$:

$$(\partial_t + (\nabla_{\vec{p}} H_i) \nabla_{\vec{r}} - (\nabla_{\vec{r}} H_i) \nabla_{\vec{p}}) f_i(\vec{r}, t, \vec{p}) = C[f_i, f_j, \dots]$$

- index i represents particle species (N, Δ , π , ρ , ...)
⇒ one equ. for each species
- Hamiltonian H_i :
 - hadronic mean fields (Skyrme-like or RMF)
 - Coulomb, “off-shell potential”
- collision term C :
 - decays and scattering processes (2- and 3-body)
 - low energy: resonance model, high energy: string fragment.
- all equations coupled through collision term and mean fields
- semi-classical: not all quantum effects included

DEGREES OF FREEDOM: HADRONS!

- GiBUU is a purely hadronic model
- no partonic phase
- leptons usually not 'transported' (but: initial eN , νN , γN interactions, leptonic and photonic decays)
- GiBUU includes a good part of the whole hadronic zoo (currently 61 baryons and 22 mesons)
- most known states made of of u,d,s,c quarks included (currently no b)
- in principle we need:
 - cross sections for collisions among all of them, at all possible energies (often not known precisely or no data available)
 - mean-field potentials for each particle species (often also not well-known)
- often experiments are set up to measure these quantities (examples: ωN cross section, Kaon potential, ...)
- \Rightarrow put some hypothesis into model and compare to data

PARTICLE SPECIES AND ID CODES

particle	mass	width	GiBUU ID	PDG IDs
N	0.983	0	1	p=2212, n=2112
Δ	1.232	0.118	2	2224, 2214, 2114, 1114
N^*			3-18	
Δ^*			19-31	
Λ	1.116	0	32	3122
Σ	1.189	0	33	3222,3212,3112
Λ^*, Σ^*			34-52	
π	0.138	0	101	$\pi^+ = 211, \pi^0 = 111, \pi^- = -211$
η	0.547		102	
ρ	0.775	0.149	103	213,113,-213
σ			104	
ω	0.782	0.004	105	
η'	0.957		106	
K	0.496	0	110	$K^+ = 321, K^0 = 311$
\bar{K}	0.496	0	111	$K^- = -321, \bar{K}^0 = -311$

... and more. Full list at:

<https://gibuu.hepforge.org/trac/wiki/ParticleIDs>

- idea: approximate full phase-space distribution by a sum of Delta-functions:

$$f(\vec{r}, t, \vec{p}) \propto \sum_i \delta(\vec{r} - \vec{r}_i(t)) \delta(\vec{p} - \vec{p}_i(t))$$

- each Delta-function represents one particle with a sharp position and momentum
- mental picture: particles as 'billard balls' (behaving almost like classical particles)
- sufficiently large number of test particles needed to obtain a good approximation

- to solve the BUU equation numerically, the time axis is discretized
- collisions only happen at discrete time steps, in between: propagation through mean fields
- typical time-step size: $\Delta t = 0.2 \text{ fm}/c$
- start at $t = 0$ and run a number of time steps (N) until $t_{max} = N \cdot \Delta t$
- typically: $t_{max} \approx 20 - 100 \text{ fm}/c \quad \Rightarrow \quad N \approx 100 - 500$

MEAN-FIELD POTENTIALS

- GiBUU has can handle two types of mean-field potentials:
 - non-relativistic Skyrme-type potentials
 - relativistic mean fields (RMF)
- in the most general form a potential can enter the single-particle energy like:

$$H = \sqrt{(m + V)^2 + (\vec{p} - \vec{U})^2} + U_0$$

- RMF potential is Lorentz vector $U^\mu = (U_0, \vec{U})$
- Skyrme potential enters as U_0 only (non-rel), and is bound to a special frame (local rest frame)
- scalar potential V (\rightarrow mass shift)

$$U_0(x, \vec{p}) = A \frac{\rho}{\rho_0} + B \left(\frac{\rho}{\rho_0} \right)^\gamma + \frac{2C}{\rho_0} \sum_{i=p,n} \int \frac{gd^3p'}{(2\pi)^3} \frac{f_i(x, \vec{p}')}{1 + (\vec{p} - \vec{p}')^2/\Lambda^2} \\ + d_{\text{symm}} \frac{\rho_p(x) - \rho_n(x)}{\rho_0} \tau_i$$

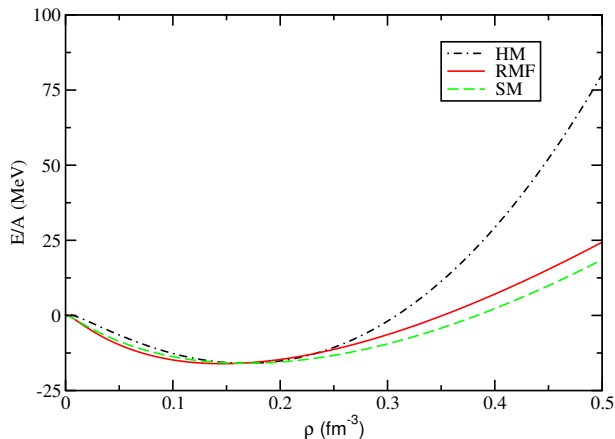
- defined in local rest frame (LRF), where spatial baryon current vanishes ($\vec{j} = 0$)
- six parameters: A , B , γ , C , Λ , d_{symm}
- fixed to
 - nuclear binding energy of 16 MeV at $\rho = \rho_0$ in isospin-sym. matter
 - nuclear-matter incompressibility $K = 200 - 380$ MeV

- proper relativistic mean-field description
- based on (nonlinear) Walecka-type Lagrangian:

$$\begin{aligned}
 \mathcal{L} = & \bar{\Psi} \left[\gamma_{\mu} (i\partial^{\mu} - g_{\omega}\omega^{\mu} - g_{\rho}\vec{\tau}\vec{\rho}^{\mu} - \frac{e}{2}(1 + \tau^3)A^{\mu}) - m_N - g_{\sigma}\sigma \right] \Psi \\
 & + \frac{1}{2}\partial_{\mu}\sigma\partial^{\mu}\sigma - U(\sigma) - \frac{1}{4}\Omega_{\mu\nu}\Omega^{\mu\nu} + \frac{1}{2}m_{\omega}^2\omega^2 - \frac{1}{4}\vec{R}_{\mu\nu}\vec{R}^{\mu\nu} \\
 & + \frac{1}{2}m_{\rho}^2\vec{\rho}^2 - \frac{1}{16\pi}F_{\mu\nu}F^{\mu\nu}
 \end{aligned}$$

- theoretically cleaner, but computationally more demanding

EQUATION OF STATE



- HM: hard momentum-dependent Skyrme
- SM: soft momentum-dependent Skyrme
- RMF: relativistic mean-fields (Walecka)

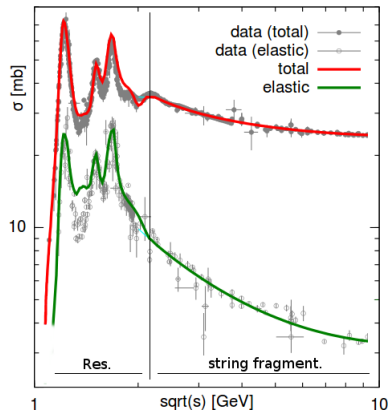
- contains one-, two- and three-body collisions

$$C = C_{1 \rightarrow X} + C_{2 \rightarrow X} + C_{3 \rightarrow X} + \dots$$

- $C_{1 \rightarrow X}$: resonance decays
- $C_{2 \rightarrow X}$: proper two-body collisions
 - elastic and inelastic
 - any number of particles in final state: $X \geq 1$
 - baryon-meson, baryon-baryon, meson-meson
- $C_{3 \rightarrow X}$: three-body coll. (only relevant at high densities)
- at low energies: cross sections based on resonance excitations (e.g. $\pi N \rightarrow N^*$, $NN \rightarrow NN^*$)
- at high energies: string fragmentation (hard parton scattering + formation and decay of 'flux tube')

BARYON-MESON COLLISIONS

- πN scattering cross section shows clear resonance peaks below $\sqrt{s} \approx 2$ GeV
- “resonance regime”: excitation of N^* and Δ^* states
- cross section obtained by Breit-Wigner formula
- above 2 GeV: cross section is flat, different mechanism (non-resonant)
- typically described by string-fragmentation models (e.g. Pythia)
- in GiBUU: transition between resonance model and string frag. at $\sqrt{s} = 2.2 \pm 0.2$ GeV



RESONANCE MODEL

- all resonance parameters, decays modes and width parametriz. taken from: Manley/Saleski, Phys. Rev. D 45 (1992)
- Manley: PWA of $\pi N \rightarrow \pi N$ and $\pi N \rightarrow 2\pi N$ data
- important point: **consistency!** (coupled-channel constraints)

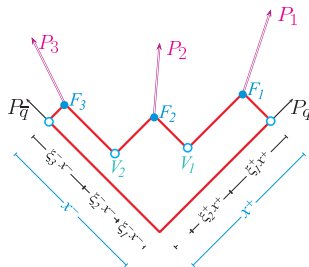
	rating	M_0	Γ_0	$ \mathcal{M}^2 /16\pi$ [mb GeV ²]		branching ratio in %						
		[MeV]	[MeV]	NR	ΔR	πN	ηN	$\pi\Delta$	ρN	σN	$\pi N^*(1440)$	$\sigma\Delta$
P ₁₁ (1440)	****	1462	391	70	—	69	—	22 _P	—	9	—	—
S ₁₁ (1535)	***	1534	151	8	60	51	43	—	2 _S + 1 _D	1	2	—
S ₁₁ (1650)	****	1659	173	4	12	89	3	2 _D	3 _D	2	1	—
D ₁₃ (1520)	****	1524	124	4	12	59	—	5 _S + 15 _D	21 _S	—	—	—
D ₁₅ (1675)	****	1676	159	17	—	47	—	53 _D	—	—	—	—
P ₁₃ (1720)	*	1717	383	4	12	13	—	—	87 _P	—	—	—
F ₁₅ (1680)	****	1684	139	4	12	70	—	10 _P + 1 _F	5 _P + 2 _F	12	—	—
P ₃₃ (1232)	****	1232	118	OBE	210	100	—	—	—	—	—	—
S ₃₁ (1620)	**	1672	154	7	21	9	—	62 _D	25 _S + 4 _D	—	—	—
D ₃₃ (1700)	*	1762	599	7	21	14	—	74 _S + 4 _D	8 _S	—	—	—
P ₃₁ (1910)	****	1882	239	14	—	23	—	—	—	—	67	10 _P
P ₃₃ (1600)	***	1706	430	14	—	12	—	68 _P	—	—	20	—
F ₃₅ (1905)	***	1881	327	7	21	12	—	1 _P	87 _P	—	—	—
F ₃₇ (1950)	****	1945	300	14	—	38	—	18 _F	—	—	—	44 _F

$$\Gamma_{R \rightarrow ab}(m) = \Gamma_{R \rightarrow ab}^0 \frac{\rho_{ab}(m)}{\rho_{ab}(M^0)}$$

$$\rho_{ab}(m) = \int dp_a^2 dp_b^2 \mathcal{A}_a(p_a^2) \mathcal{A}_b(p_b^2) \frac{\rho_{ab}}{m} B_{L_{ab}}^2(p_{ab} R) \mathcal{F}_{ab}^2(m)$$

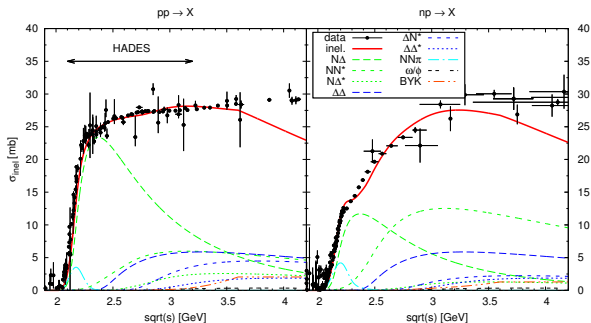
STRING FRAGMENTATION: PYTHIA

- idea: hard qq scattering (pQCD) creates a color flux tube ('string'), which then fragments into hadrons (via $q\bar{q}$ pair production)
- high-energy model (10 GeV ... TeV)
- breaks down at low energies
- popular implementation: PYTHIA (Lund string model)
- includes a few resonances (Δ , ρ , ...) but not all the higher states (N^* , Δ^*)
- phenomenological 'fragmentation function' determines when and how the string breaks
- parameters fitted to reproduce data (may not be universal, different 'tunes' available)



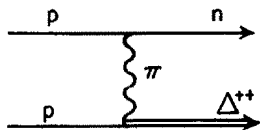
BAR-BAR COLLISIONS

- also for NN collisions: low-energy regime dominated by resonance production
- but: no nice peaks due to two-body kinematics
- not fully clear how far resonance approach will work
- $NN \rightarrow NR, \Delta R$ ($R = \Delta, N^*, \Delta^*$)
- all π, η and ρ mesons produced via R decays (ω, ϕ : non-res.)
- good descr. of total NN cross sections up to $\sqrt{s} \approx 3.4\text{GeV}$



RESONANCE PRODUCTION

- $NN \rightarrow N\Delta$: OBE model [Dmitriev et al, NPA 459 (1986)]



- other resonances produced via phase-space approach (constant matrix elements), Teis Z. Phys. A 356, 1997:

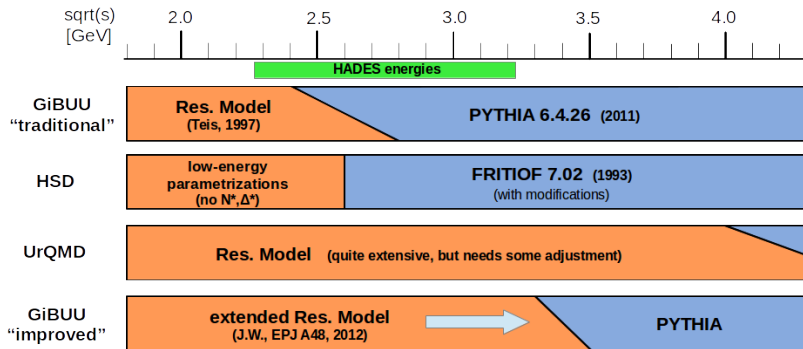
$$\sigma_{NN \rightarrow NR} = \frac{C_I}{p_{iS}} \frac{|\mathcal{M}_{NR}|^2}{16\pi} \int d\mu \mathcal{A}_R(\mu) p_F(\mu)$$

$$\sigma_{NN \rightarrow \Delta R} = \frac{C_I}{p_{iS}} \frac{|\mathcal{M}_{\Delta R}|^2}{16\pi} \int d\mu_1 d\mu_2 \mathcal{A}_\Delta(\mu_1) \mathcal{A}_R(\mu_2) p_F(\mu_1, \mu_2)$$

- lately: introduced angular distributions $d\sigma/dt = b/t^a$, exponents a determined from HADES data

NN COLLISIONS IN DIFFERENT MODELS

- baryon-baryon collisions at low energies:
resonance models vs. string fragmentation



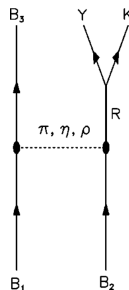
- HADES energy range is in the resonance regime
- we need one consistent model for the whole energy range!

STRANGENESS PRODUCTION

- at low energies, Kaons production probably also dominated by res. formation
- but: BRs not so well known, no Kaon data included in Manley analysis
- therefore: GiBUU does not produce Kaons in two-step processes $NN \rightarrow NN^* \rightarrow NYK$, but in direct three-body production $NN \rightarrow NYK$
- cross sections originally calculated by Tsushima et al. in eff. Lagrangian model via resonance excitation
- parametrization of these cross sections used in GiBUU:

$$\sigma(BB \rightarrow BYK) = a \left(\frac{s}{s_0} - 1 \right)^b \left(\frac{s}{s_0} \right)^c$$

$B = N, \Delta$; $Y = \Lambda, \Sigma$; s_0 : threshold; a, b, c : (free) parameters



STRANGENESS CHANNELS

No.	Reaction	s_0 (GeV ²)	a (mb)	b	c
1	$pp \rightarrow p\Lambda K^+$	6.504	1.879	2.176	5.264
2	$pn \rightarrow n\Lambda K^+$	6.504	2.812	2.121	4.893
3	$pp \rightarrow p\Sigma^0 K^+$	6.904	5.321	2.753	8.510
4	$nn \rightarrow n\Sigma^- K^+$	6.904	7.079	2.760	8.164
5	$pn \rightarrow n\Sigma^0 K^+$	6.904	6.310	2.773	7.820
6	$np \rightarrow p\Sigma^- K^+$	6.904	11.02	2.782	7.674
7	$pp \rightarrow n\Sigma^+ K^+$	6.904	1.466	2.743	3.271
8	$pp \rightarrow \Delta^- \Lambda K^+$	8.085	6.166	2.842	1.960
9	$pp \rightarrow \Delta^+ \Sigma^- K^+$	8.531	10.00	2.874	2.543
10	$\Delta^+ n \rightarrow p\Lambda K^+$	6.504	8.337	2.227	2.511
11	$\Delta^- p \rightarrow n\Sigma^- K^+$	6.904	52.72	2.799	6.303
12	$\Delta^{++} p \rightarrow \Delta^+ \Lambda K^+$	8.085	2.704	2.303	5.551
13	$\Delta^+ n \rightarrow \Delta^0 \Lambda K^+$	8.085	0.312	2.110	2.165
14	$\Delta^+ p \rightarrow \Delta^+ \Lambda K^+$	8.085	2.917	2.350	6.557
15	$\Delta^+ n \rightarrow \Delta^+ \Sigma^- K^+$	8.531	10.33	2.743	8.915
16	$\Delta^0 p \rightarrow \Delta^+ \Sigma^- K^+$	8.531	2.128	2.843	5.986
17	$\Delta^+ n \rightarrow \Delta^+ \Sigma^- K^+$	8.531	10.57	2.757	10.11
18	$\Delta^{++} p \rightarrow \Delta^+ \Sigma^0 K^+$	8.531	10.30	2.748	9.321
19	$\Delta^+ n \rightarrow \Delta^0 \Sigma^0 K^+$	8.531	1.112	2.846	5.943
20	$\Delta^+ p \rightarrow \Delta^+ \Sigma^0 K^+$	8.531	10.62	2.759	10.20
21	$\Delta^+ p \rightarrow \Delta^0 \Sigma^+ K^+$	8.531	0.647	2.830	3.862
22	$\Delta^+ \Delta^+ \rightarrow \Delta^+ \Lambda K^+$	8.085	1.054	2.149	7.969
23	$\Delta^0 \Delta^+ \rightarrow \Delta^+ \Lambda K^+$	8.085	0.881	2.150	7.977
24	$\Delta^0 \Delta^+ \rightarrow \Delta^0 \Lambda K^+$	8.085	0.291	2.148	7.934
25	$\Delta^+ \Delta^0 \rightarrow \Delta^+ \Sigma^- K^+$	8.531	3.532	2.953	12.06
26	$\Delta^- \Delta^0 \rightarrow \Delta^- \Sigma^- K^+$	8.531	7.047	2.952	12.05
27	$\Delta^0 \Delta^+ \rightarrow \Delta^+ \Sigma^0 K^+$	8.531	2.931	2.952	12.03
28	$\Delta^- \Delta^+ \rightarrow \Delta^0 \Sigma^- K^+$	8.531	5.861	2.952	12.04

- large number of channels!
- Tsushima parameters not always fully compatible with data
- some had to be refitted to HADES data recently

- dileptons are rare probes: $BR(h \rightarrow e^+e^-) \approx 10^{-5}$
- special treatment is necessary: “shining” approach
- hadrons continuously emit ('virtual') lepton pairs, those are 'weighted down' to fit the BR
- leptons not 'propagated', rescattering with hadrons neglected
- dilepton production has to be turned on separately
- included channels:
 - $V \rightarrow e^+e^-$ (with $V = \rho, \omega, \phi$)
 - $P \rightarrow \gamma e^+e^-$ (with $P = \pi^0, \eta, \eta'$)
 - $\omega \rightarrow \pi^0 e^+e^-$
 - $\Delta \rightarrow Ne^+e^-$
 - $R \rightarrow \rho N \rightarrow Ne^+e^-?$
 - Bremsstrahlung in soft-photon approximation

references for further details:

- 'GiBUU review paper':
O. Buss et al., Phys. Rept. 512 (2012) 1-124
- many specialized papers and theses:
<https://gibuu.hepforge.org/trac/wiki/Paper>
- documentation on website
- code itself!

end of general intro

questions?