Hadronization in eA collisions with GiBUU

K. Gallmeister for the GiBUU group
Goethe-Universität, Frankfurt

motivation

kinetic theory, GiBUU

EMC & Hermes & CLAS

Bundesministerium
für Bildung
und Forschung

UTFSM Seminar

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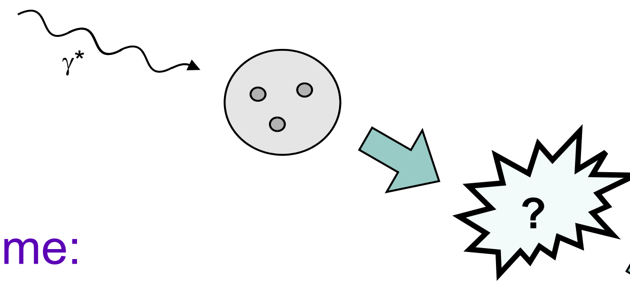
Motivation

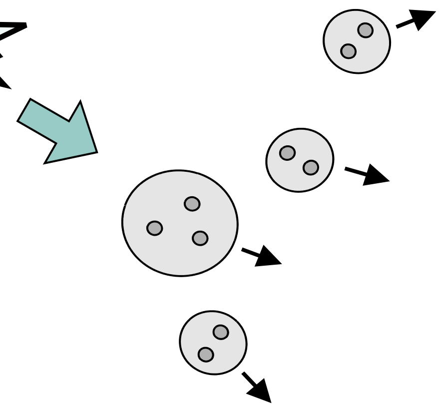
- elementary reactions (*eN*, γ*N*) on nucleon:



formation time:
estimation via hadronic radius

$$\tau_F \geq \frac{r_h}{c} \qquad = 0.5 \cdots 0.8\,\mathrm{fm}$$

time dilatation: $t_F = \gamma\tau_F \qquad (\sim 10\,\mathrm{fm})$



reaction products hadronize long before they reach the detector

- nuclear reactions (*eA*, γ*A* @ GeV energies) :

interactions with nuclear medium during formation

→ space-time picture of hadronization $\qquad \sigma^*/\sigma_H \;\sim\; t^{0,1,2,\ldots}$

Observables, Experiments

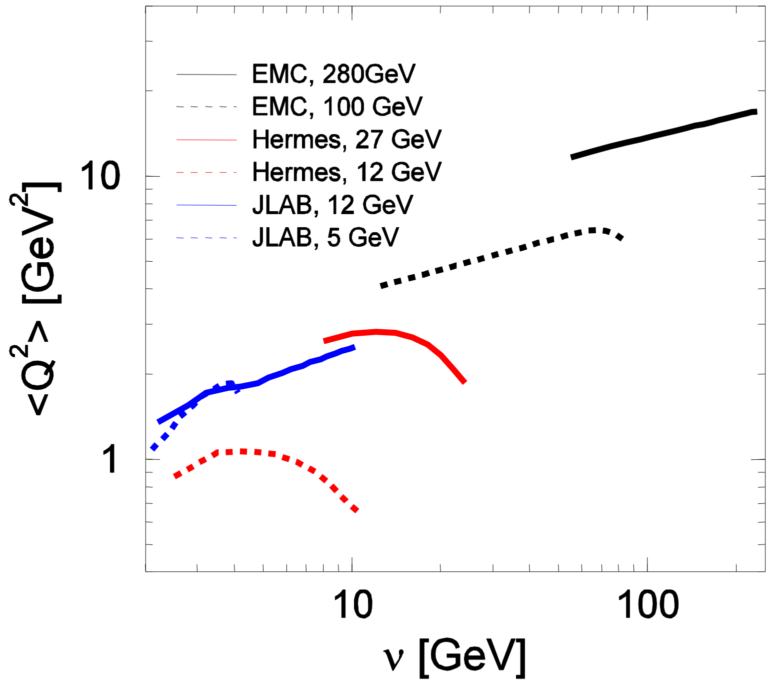
- $$R^h(z_h,\ldots) = \frac{\left.\frac{N_h(z_h,\ldots)}{N_e(\ldots)}\right|_A}{\left.\frac{N_h(z_h,\ldots)}{N_e(\ldots)}\right|_D}$$
- $$\Delta p_T^2 = \langle p_T^2\rangle_A - \langle p_T^2\rangle_D$$

- hadronic: $z_h = \frac{E_h}{\nu}\ ,\quad p_T\ ,\cdots$
- photonic: $\nu\ ,\ Q^2\ ,\ W\ ,\ x_B\ ,\ldots$

Experiments

	E_{lepton} =
EMC	**100…280 GeV**
Hermes	**27 GeV**
	12 GeV
CLAS	12 GeV (upgrade)
	5 GeV
EIC	e.g. 3+30 GeV

…multiple combinations of targets

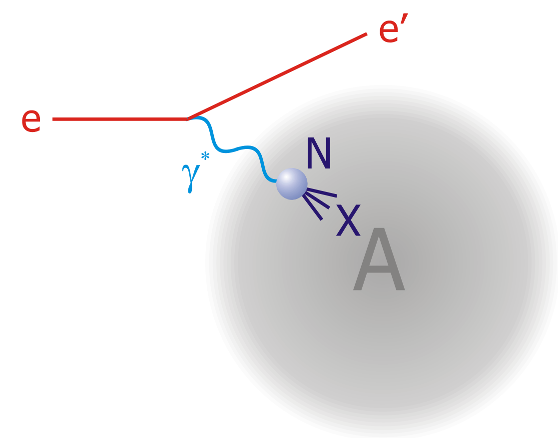


Model

- $\gamma^*N \rightarrow X$ using PYTHIA

    additional:
    - binding energies
    - Fermi motion
    - Pauli blocking
    - coherence length effects

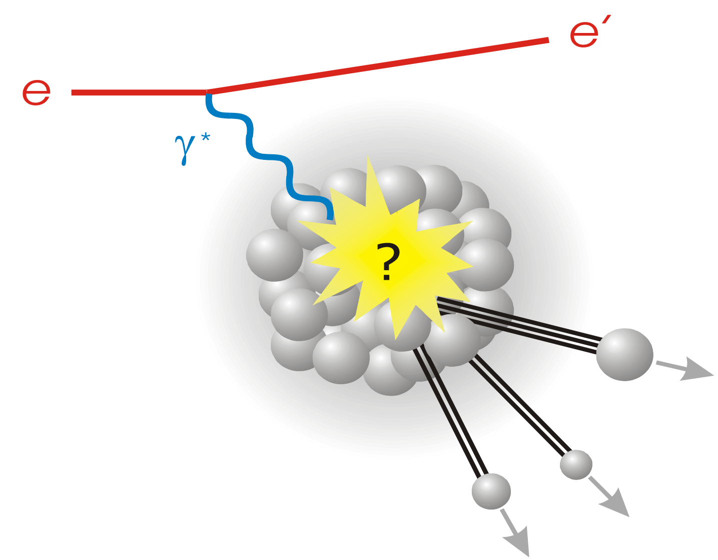
        extended for exclusive channels



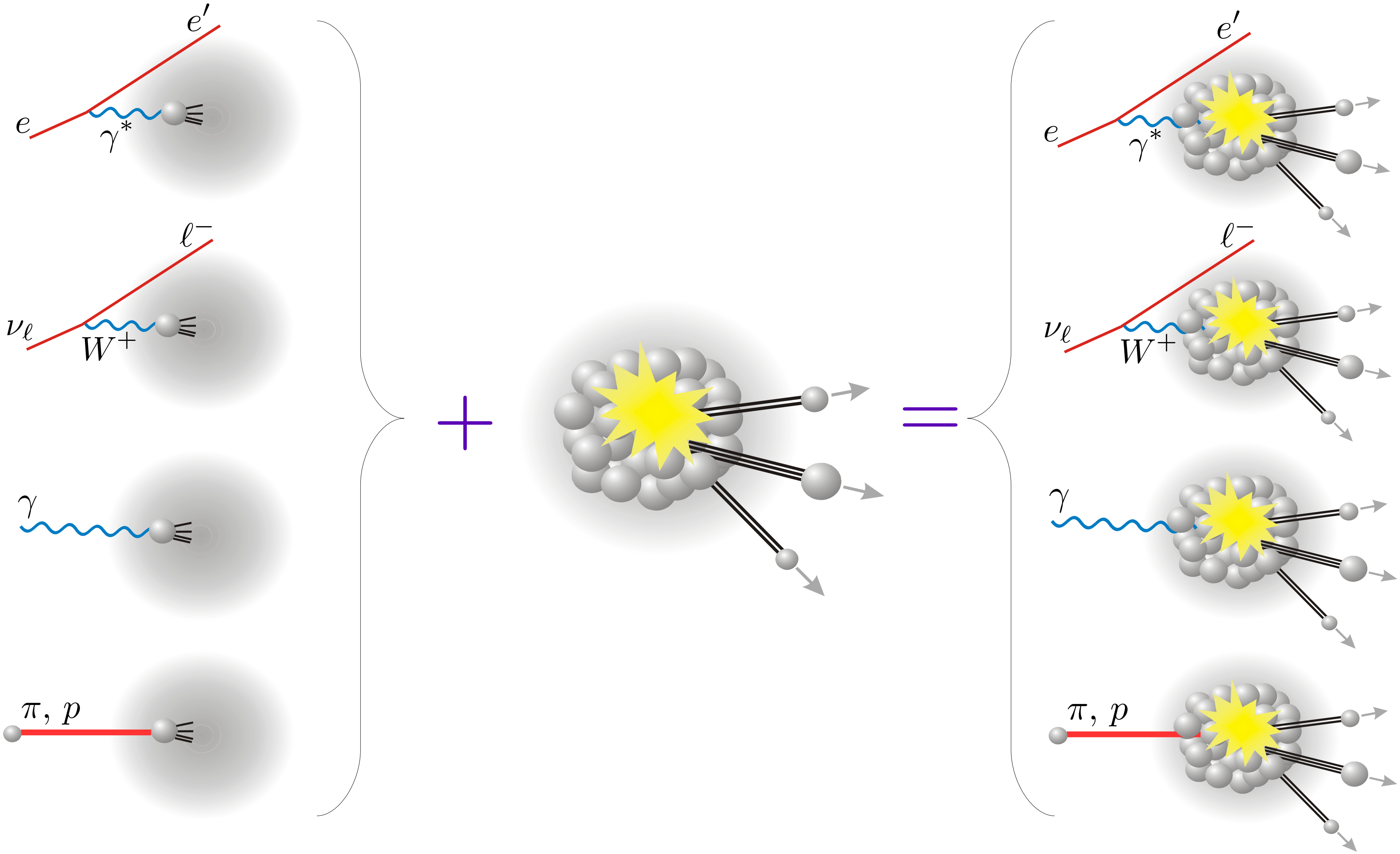
- propagation of final state X within GiBUU transport model

    http://gibuu.hepforge.org

    - elastic/inelastic scatterings (coupled channels)
    - experimental acceptance



GiBUU = plug-in system



init + FSI = full event

GiBUU: The FSI transport

Some kinetic theory

- **distribution function** $f(x,p) \qquad x = (t, \vec{x}\,),\ \ p = (E, \vec{p}\,)$

  describes (density) distribution of (single) particles

  number of particles in a given phase-space volume: $\Delta N = f(x,p)\,\Delta^3 x\,\Delta^3 p$

- for each particle species: $f_N\,,\ \ f_\pi\,,\ \ f_\Delta\,,\ \ ,\ldots$

- **continuity equation** for free, non-interacting particles

$$p^\mu \partial_\mu\, f(x,p) = 0$$

straight line propagation of particles, no collisions

- adding external forces (mean field potentials): **Vlasov eq.**

$$[\partial_t + (\nabla_p E)\nabla_r - (\nabla_r E)\nabla_p]\, f(x,p) = 0$$

propagation through mean field, no collisions

Adding collisions

(...forget about mean fields, but add collisions…)

- continuity eq. + collision term → **Boltzmann eq.**

$$p^\mu \partial_\mu f(x,p) = C(x,p)$$

collision integral has gain and loss term:

$$C(x,p) = C_{\text{gain}}(x,p) + C_{\text{loss}}(x,p)$$

- mean fields and collision term:

Boltzmann-Uehling-Uhlenbeck eq. (BUU or VUU)

$$\left[\partial_t + (\nabla_p H_i)\nabla_r - (\nabla_r H_i)\nabla_p\right] f_i(\vec{r},t,\vec{p}) = C\left[f_i, f_j, \ldots\right]$$

The BUU equation

$$\left[\partial_t + (\nabla_p H_i)\nabla_r - (\nabla_r H_i)\nabla_p\right] f_i(\vec{r},t,\vec{p}) = C\left[f_i, f_j, \ldots\right]$$

- describes space-time evolution of single particle densities
- index i represents particle species
  $\rightarrow$ one equation for each species

$$i = N, \Delta, \pi, \rho, \ldots$$

- Hamiltonian H_i
  - hadronic mean fields (Skyrme/Welke or RMF)
  - Coulomb
  - „off-shell-potential“
- collision term C
  - decay and scattering processes: 1-, 2- and 3-body
  - (low energy: resonance model, high energy: string model)
  - contains Pauli-blocking
- equations coupled via mean fields and via collision term

Degrees of Freedom

- GiBUU is purely hadronic (no partonic phase)

- **leptons**: not ‚transported‘, but:
  - e+N, nu+N, gamma+N initial events
  - leptonic/photonic decays

- 61 **baryons**, 22 **mesons**

  (strangeness and charm included, no bottom)

  properties from Manley analysis, PDG for strange&charm

- in principle one needs:
  - cross sections for collisions between all of them (all energies)
  - mean-field potentials for all species

  often not known, thus use hypothesis/models/guesses

Particle species

- important particles:

particle	mass	width	GiBUU ID	PDG IDs
N	0.983	0	1	p=2212, n=2112
Δ	1.232	0.118	2	2224, 2214, 2114, 1114
N^*			3-18	
Δ^*			19-31	
Λ	1.116	0	32	3122
Σ	1.189	0	33	3222,3212,3112
Λ^*, Σ^*			34-52	
π	0.138	0	101	$\pi^+ = 211,\ \pi^0 = 111,\ \pi^- = -211$
η	0.547		102	
ρ	0.775	0.149	103	213,113,-213
σ			104	
ω	0.782	0.004	105	
η'	0.957		106	
K	0.496	0	110	$K^+ = 321,\ K^0 = 311$
$\bar{K}$	0.496	0	111	$K^- = -321,\ \bar{K}^0 = -311$

https://gibuu.hepforge.org/trac/wiki/ParticleIDs

Mean-field potentials

- two types of mean-field potentials:
  - non-relativistic Skyrme-type potentials
  - relativistic mean fields (RMF)

- potential may enter single-particle energy as

$$H = \sqrt{(m+V)^2 + (\vec{p} + \vec{U}\,)^2} + U_0$$

- RMF is Lorentz vector U^μ
- Skyrme enters as U_0, bound to specific frame (LRF)

- Scalar Potential V: mass shift

Collision term

- contains one-, two-, and three-body collisions

$$C = C_{1\to X} + C_{2\to X} + C_{3\to X}$$

(1) resonance decays

(2) two-body collisions

- elastic and inelastic
- any number of particles in final state
- baryon-meson, baryon-baryon, meson-meson

(3) three-body collisions (only relevant at high densities)

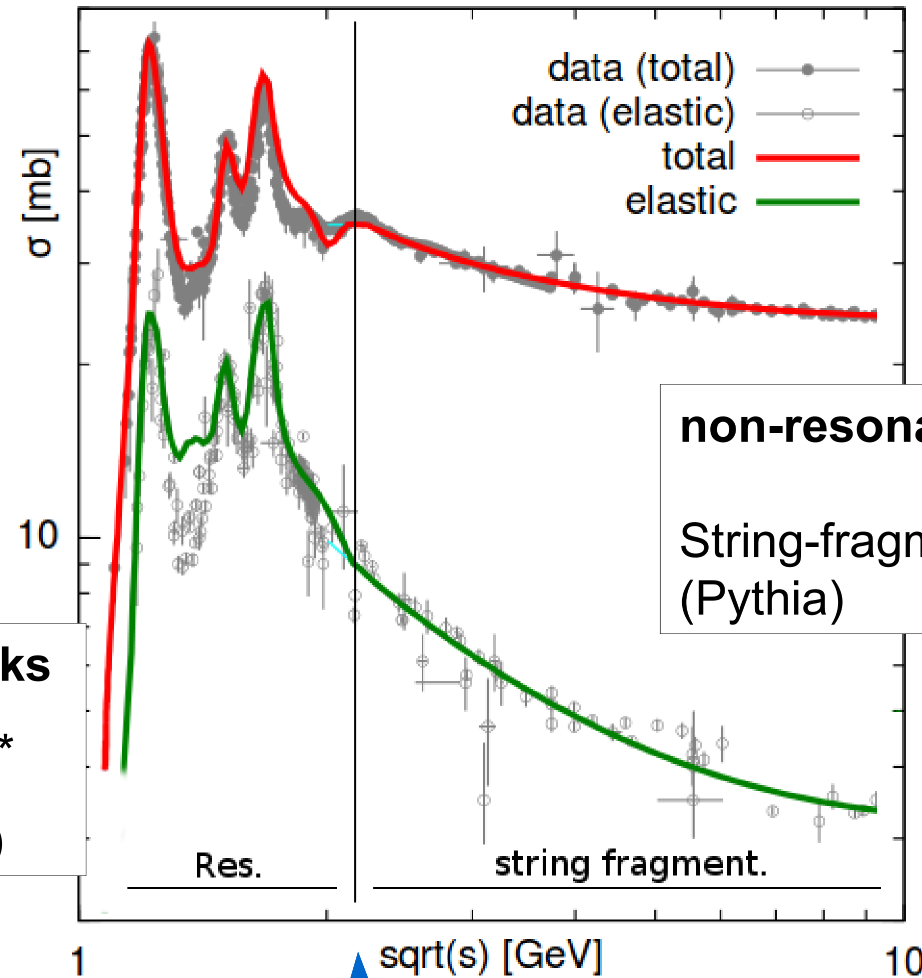
- **low energies:** cross sections based on resonances

$$\text{e.g. } \pi N \to N^*, \; NN \to NN^*$$

- **high energies:** string fragmentation

Baryon-Meson collisions

- example: πN cross section



clear resonance peaks

excitation of N* and Δ*

(Breit-Wigner shapes)

non-resonant

String-fragmentation (Pythia)

$$\mathcal{A}(p) = \frac{1}{\pi} \frac{\sqrt{p^2}\Gamma}{(p^2 - M_0^2)^2 + p^2\Gamma^2}$$

$$\sqrt{s} = 2.2 \pm 0.2\,\mathrm{GeV}$$

Collision term

- 2-to-2 term $\quad (12 \leftrightarrow 1'2')$

$$
\begin{aligned}
&C^{(2,2)}(x,p_1)\\
&= C^{(2,2)}_{\text{gain}}(x,p_1) - C^{(2,2)}_{\text{loss}}(x,p_1)\\
&= \frac{\mathcal{S}_{1'2'}}{2p_1^0 g_{1'} g_{2'}} \int \frac{\mathrm{d}^4 p_2}{(2\pi)^4 2p_2^0} \int \frac{\mathrm{d}^4 p_{1'}}{(2\pi)^4 2p_{1'}^0} \int \frac{\mathrm{d}^4 p_{2'}}{(2\pi)^4 2p_{2'}^0}\\
&\quad \times (2\pi)^4 \delta^{(4)}(p_1 + p_2 - p_{1'} - p_{2'}) \overline{|\mathcal{M}_{12\to 1'2'}|^2}\\
&\quad \times [F_{1'}(x,p_{1'}) F_{2'}(x,p_{2'}) \overline{F}_1(x,p_1) \overline{F}_2(x,p_2)\\
&\qquad - F_1(x,p_1) F_2(x,p_2) \overline{F}_{1'}(x,p_{1'}) \overline{F}_{2'}(x,p_{2'})]
\end{aligned}
$$

- $$F(x,p) = 2\pi g\, f(x,p)\, \mathcal{A}(x,p)$$
  $$\overline{F}(x,p) = 2\pi g\, [1 - f(x,p)]\, \mathcal{A}(x,p)$$

Pauli-blocking

Testparticle ansatz

$$\left[\partial_t + (\nabla_p H_i)\nabla_r - (\nabla_r H_i)\nabla_p\right] f_i(\vec{r}, t, \vec{p}) = C\left[f_i, f_j, \ldots\right]$$

- *idea:*
  approximate full phase-space density distribution by a sum of delta-functions

$$f\left(\vec{r}, t, \vec{p}\right) \sim \sum_{i=1}^{N_{\mathrm{test}}} \delta\left(\vec{r} - \vec{r}_i(t)\right) \delta\left(\vec{p} - \vec{p}_i(t)\right)$$

- each delta-function represents one (test-)particle with a sharp position and momentum

- large number of test particles needed

Ensemble techniques

- **“full ensembles” technique**
- every testparticle may interact with every other one
- rescaling of cross section

$$\sigma_{ij} \to \frac{1}{N_{\text{test}}}\sigma_{ij}$$

- Pros:
  - locality of collisions
- Cons:
  - calculational time: collisions scale with $(N_{\text{test}})^2$
  - energy not conserved per ensemble, on average only
  - conserved quantum numbers are respected on average only (‘canonical’)

$$\underbrace{K}_{\text{ensemble } i} + \underbrace{\overline{K}}_{\text{ensemble } j} \to \pi\pi$$

Ensemble techniques

- **“parallel ensembles” technique**
- *idea:*

  testparticle index is also ensemble index

- N_{test} independent runs, densities etc. may be averaged

- Pros:
  - calculational time: collisions scale with N_{test}
  - conserved quantum numbers are strictly respected (‘microcanonical’)
- Cons:
  - non-locality of collisions $\sigma_{ij} \simeq 30\,\mathrm{mb} \ \rightarrow \ r = 1\,\mathrm{fm}$

Time evolution

- time axis is discretized
  - collisions only happen at discrete time steps,
  - between collisions: propagation (through mean fields)

- start at $t{=}0$ and run N timesteps until $t_{\max}$
- typically:

  time step size: $\Delta t = 0.1\text{-}0.2\,\mathrm{fm}/c$

$$N_{\text{step}}\,\Delta t = t_{\max} \approx 20\text{-}50\,\mathrm{fm}/c$$

$$\Longrightarrow \;\; N_{\text{step}} \approx 100\text{-}1000$$

- densities/potentials: if not analytically, recalc at every time step

Cross section: Geometric interpretation

- particle i and particle j collide, if during timestep Δt

$$r_{ij}(t) = |\vec{r}_i(t) - \vec{r}_j(t)| \overset{!}{\leq} \frac{\sqrt{\sigma_{ij}}}{\pi}$$

- problem 1: only for 2-body collisions

- problem 2: not invariant under Lorentz-Trafos
  - different frames may lead to different ordering of collisions
  - specific frame ('calculational frame') needed

Cross section: Stochastic interpretation

- collision rate per unit phase space

massless, no $(2\pi)^3$

$$\frac{\Delta N_{\rm coll}^{2\to 2}}{\Delta t \Delta^3 x \Delta^3 p_1} = \frac{\Delta^3 p_2}{2E_1\, 2E_2} f_1 f_2 \int \frac{\mathrm{d}^3 p_1'}{2E_1'} \frac{\mathrm{d}^3 p_2'}{2E_2'} |\mathcal{M}| \delta^{(4)}(p_1 + p_2 - p_1' - p_2')$$

$$\sigma_{22} = \frac{1}{2s} \int \frac{\mathrm{d}^3 p_1'}{2E_1'} \frac{\mathrm{d}^3 p_2'}{2E_2'} |\mathcal{M}| \delta^{(4)}(p_1 + p_2 - p_1' - p_2') \qquad f_i = \frac{\Delta N_i}{\Delta^3 x \Delta^3 p}$$

- collision **probability** in unit box $\Delta^3 x$ and unit time Δt

$$P_{22} = \frac{\Delta N_{\rm coll}^{2\to 2}}{\Delta N_1 \Delta N_2} = v_{\rm rel} \sigma_{22} \frac{\Delta t}{\Delta^3 x} \qquad \left(v_{\rm rel} = \frac{s}{2E_1 E_2} \right)$$

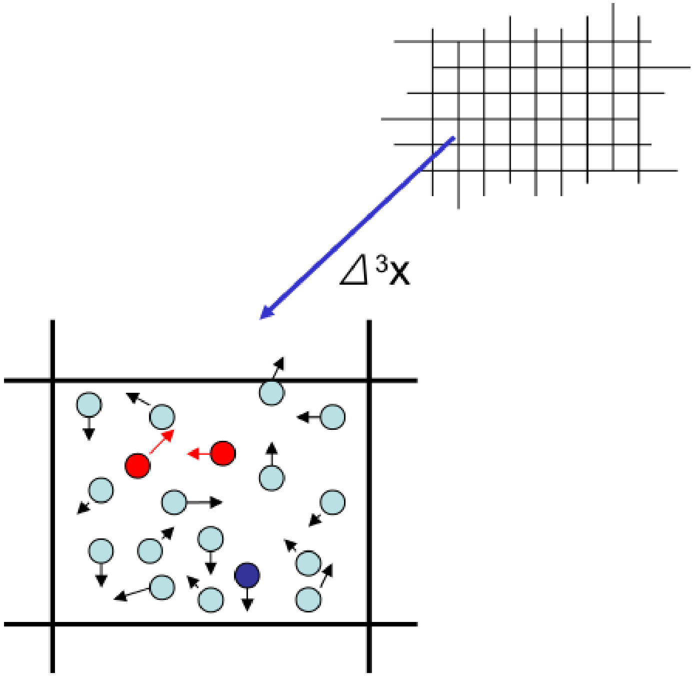
- generalisable to n-body collisions

Cross section: Stochastic interpretation

- discretize **time** and **space**

$$\mathrm{P}_{2\to X} = v_{\mathrm{rel}}\sigma_{2\to X}\frac{\Delta t}{\Delta V}$$

$$\mathrm{P}_{3\to X} = \frac{I_{3\to X}}{8E_1E_2E_3}\frac{\Delta t}{(\Delta V)^2}$$



- together with 'full ensemble'

- rescaling/speedup:

  n particles in cell

  randomly select $n/2$ pairs

  adjust probability according

$$\mathrm{P}_2 \to \frac{n(n-1)/2}{n/2}\mathrm{P}_2$$

- calculational time: collisions scale approx. with N_{test}
- labeled as *"local ensemble method"*

Init

- in principle:
  1) initialize nucleons
  2) perform **one** initial elementary event on **one** nucleon
  3) propagate nucleons and final state particles

- correct, but ‘waste of time’

- *idea:*
  final state particles do not really disturb the nucleus

- 2 particle classes:
  - ‘real particles’
  - ‘perturbative particles’

Particle classes

- 'real particles'
  - nucleons
  - may interact among each other
  - interaction products are again 'real particles'
- 'perturbative particles'
  - final state particles of initial event
  - may only interact with 'real particles'
  - interaction products are again 'perturbative particles'
- 'real particles' behave as if other particles are not there

total energy, total baryon number, etc. are **not** conserved when real and perturabtive particles are added.

Init with perturbative particles

- init
  1) initialize nucleons
  2) perform **one** initial elementary event on **every** nucleon
  3) propagate nucleons and final state particles

  - final states particles are ‘perturbative particles’
  - different final states do not interfere

- every final state particle gets a ‘perturbative weight’:
  - value: cross section of initial event
  - is inherited in every FSI

  for final spectra:

  the ‘perturbative weights’ have to be added, not only the particle numbers

Init with perturbative particles

- *idea:*

  simple workaround against oscillating ground states:

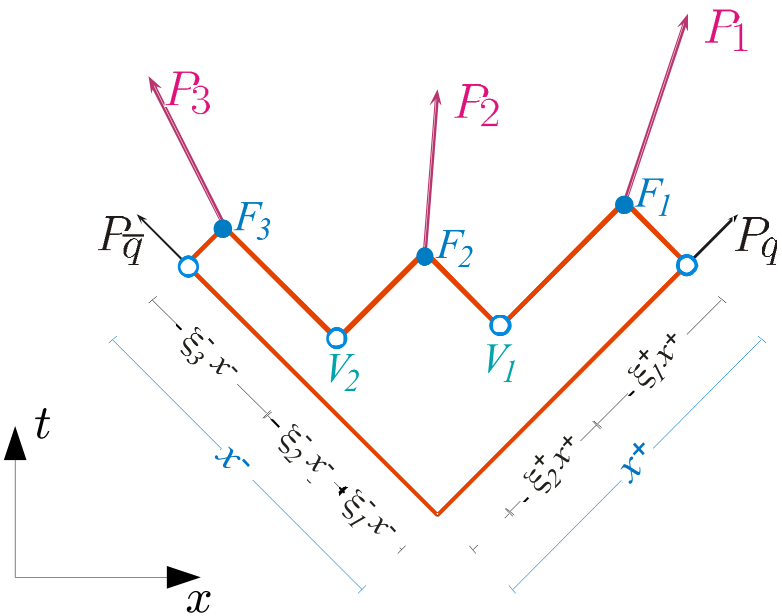
  freeze nucleon testparticles

- since nucleons are real particles, their interactions among each other should not influence final state particles

- advantage: computational time
- disadvantage: ???

GiBUU: The init

Model: Hadronization in String Model (PYTHIA/JETSET)

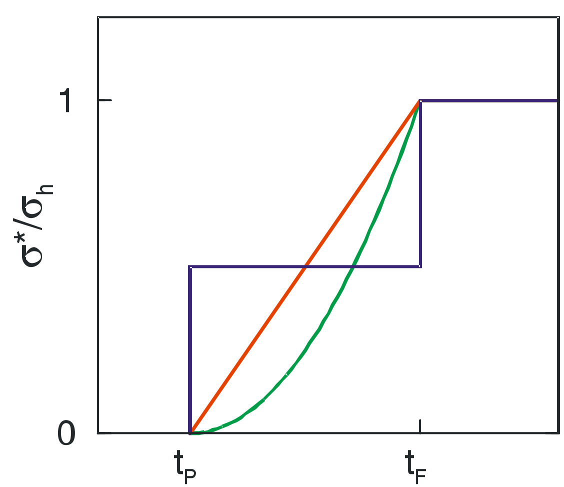


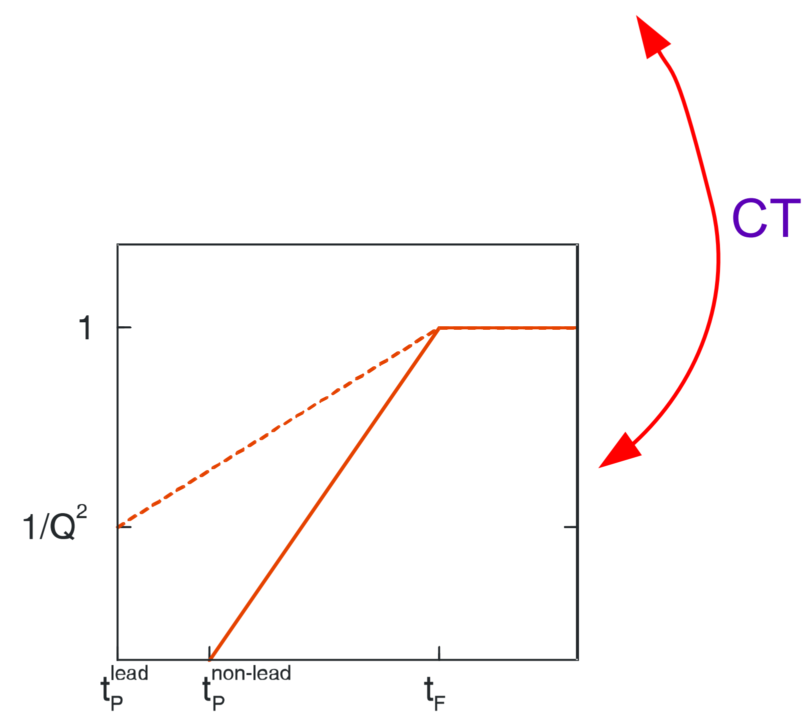
- **3 times/points per particle:**
  - „production 1“ *string-breaking*
  - „production 2“ *string-breaking*
  - „formation“ *line-meeting*

- **leading vs. non-leading**

  connection to interaction vertex

- **cross section evolution scenarios:**





Results: EMC & Hermes

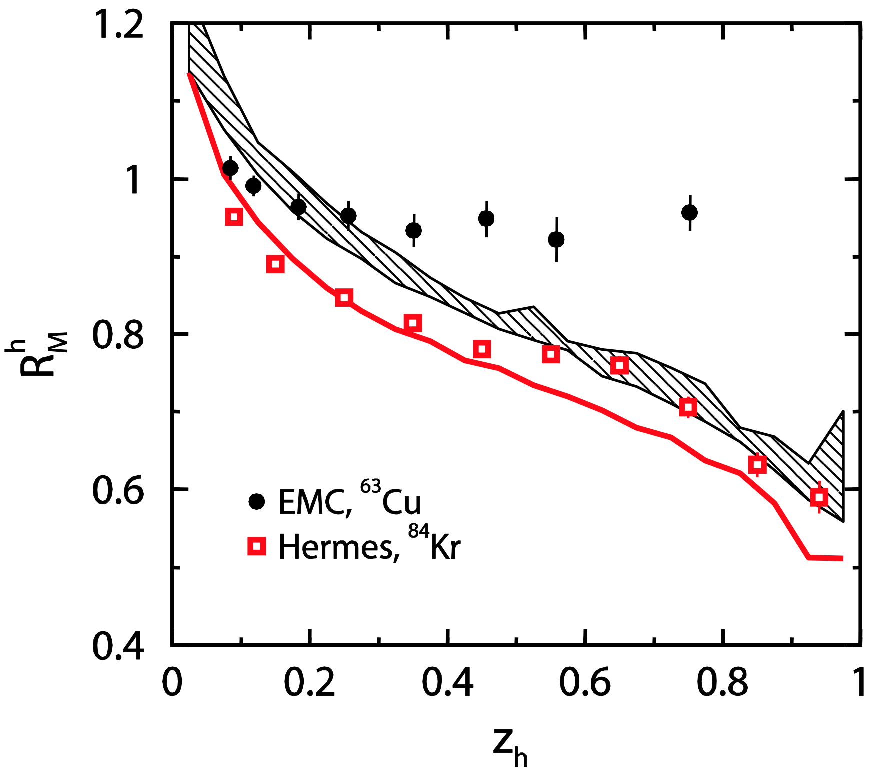
- constant cross section

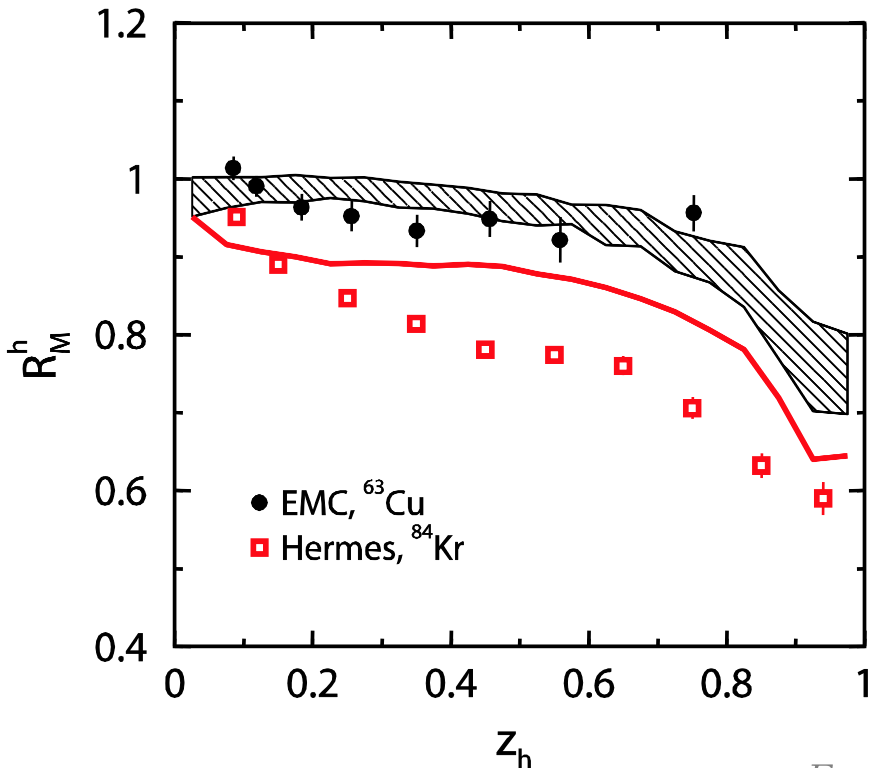
$$t = t_{\mathrm{P}} \cdots t_{\mathrm{F}} :$$

$$\sigma^* = 0.5\,\sigma_H$$

- quadratic increase

$$\sigma^* = \left(\frac{t - t_{\mathrm{P}}}{t_{\mathrm{F}} - t_{\mathrm{P}}}\right)^2 \sigma_H$$





$$z_h = \frac{E_h}{\nu}$$

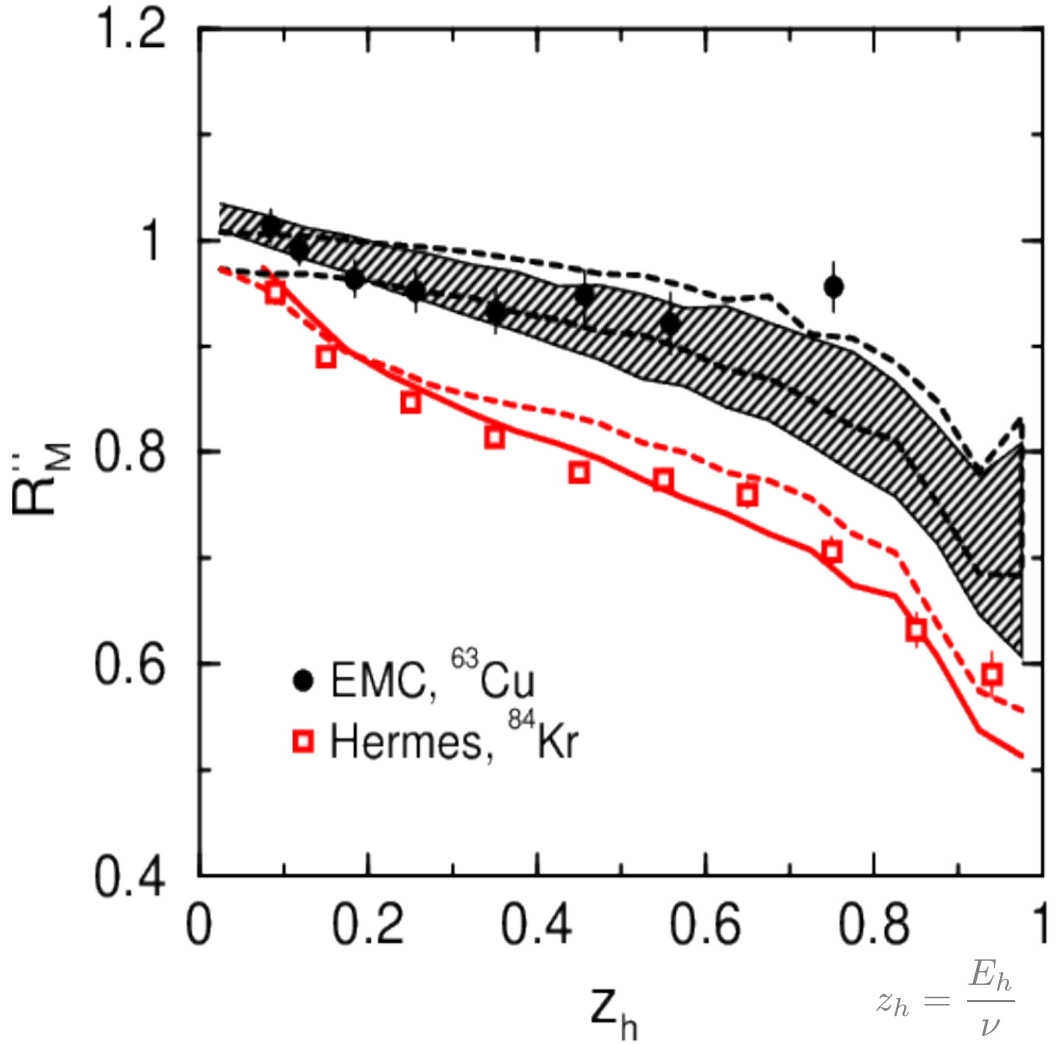
Results: EMC & Hermes

$$\frac{\sigma^*}{\sigma_H} = \frac{r_{\rm lead}}{Q^2} + \left(1 - \frac{r_{\rm lead}}{Q^2}\right)\left(\frac{t - t_{\rm P}}{t_{\rm F} - t_{\rm P}}\right)$$

!

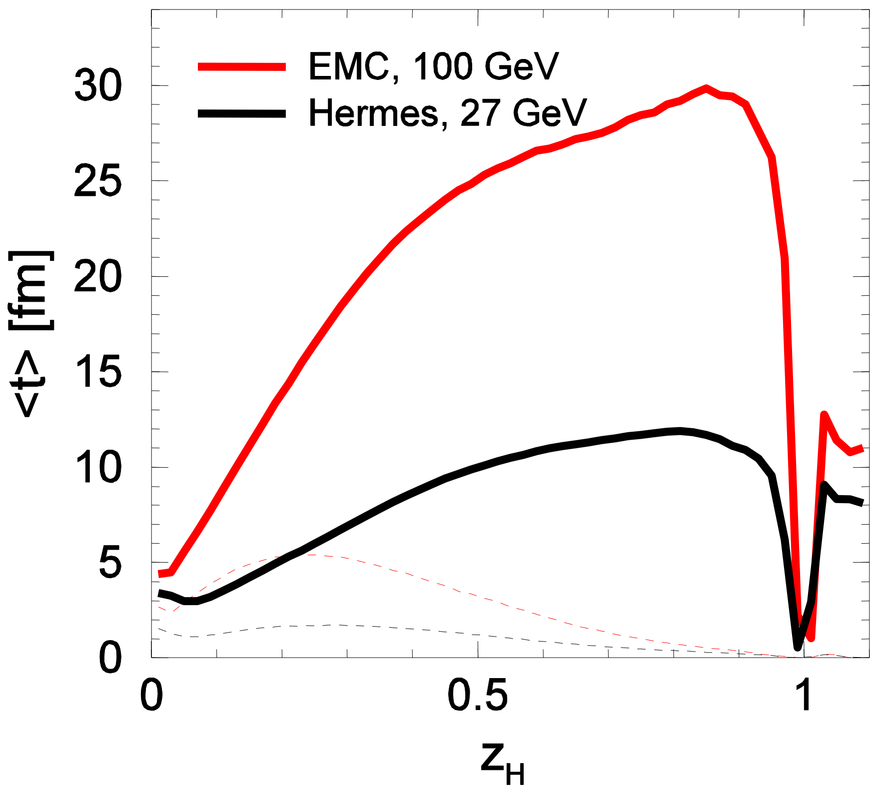
EMC@100...280 GeV
and
Hermes@27 GeV
described simultanously

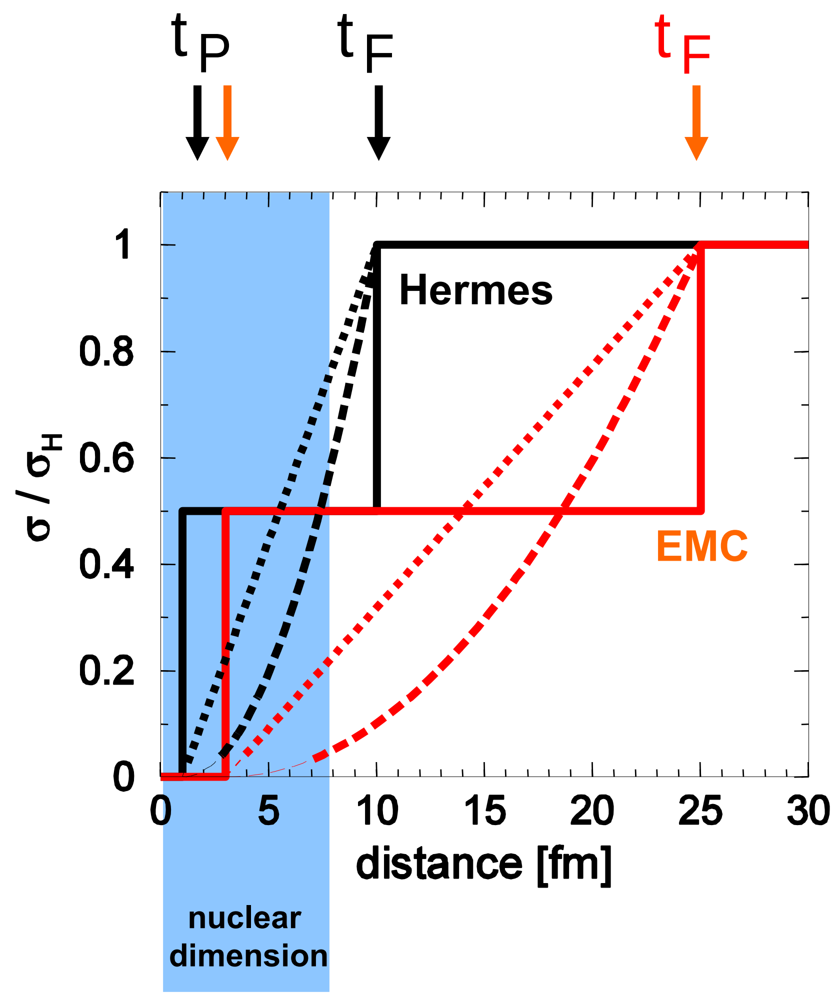
$1/Q^2$ pedestal value?
…small effect!



$z_h = \frac{E_h}{\nu}$

Times

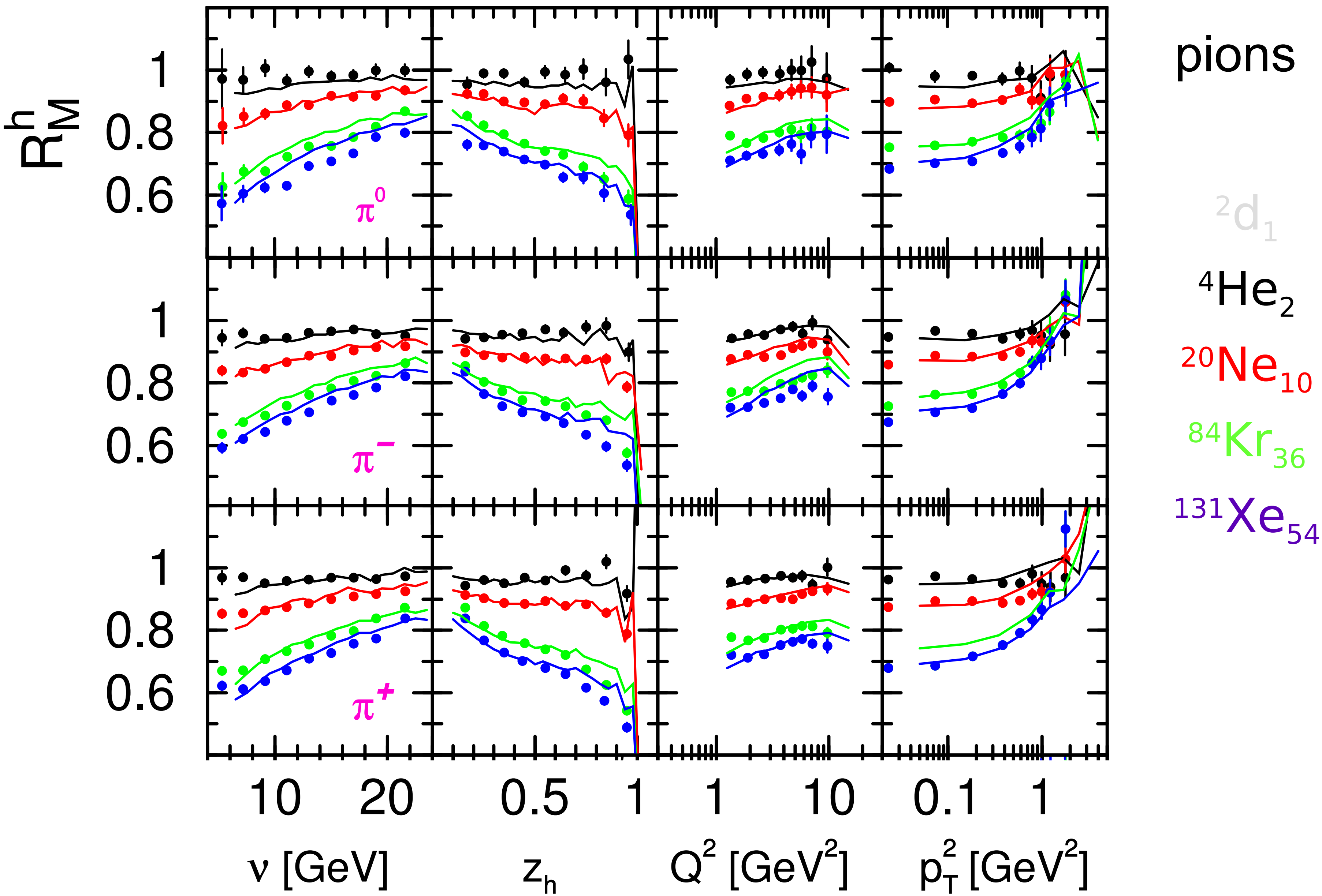




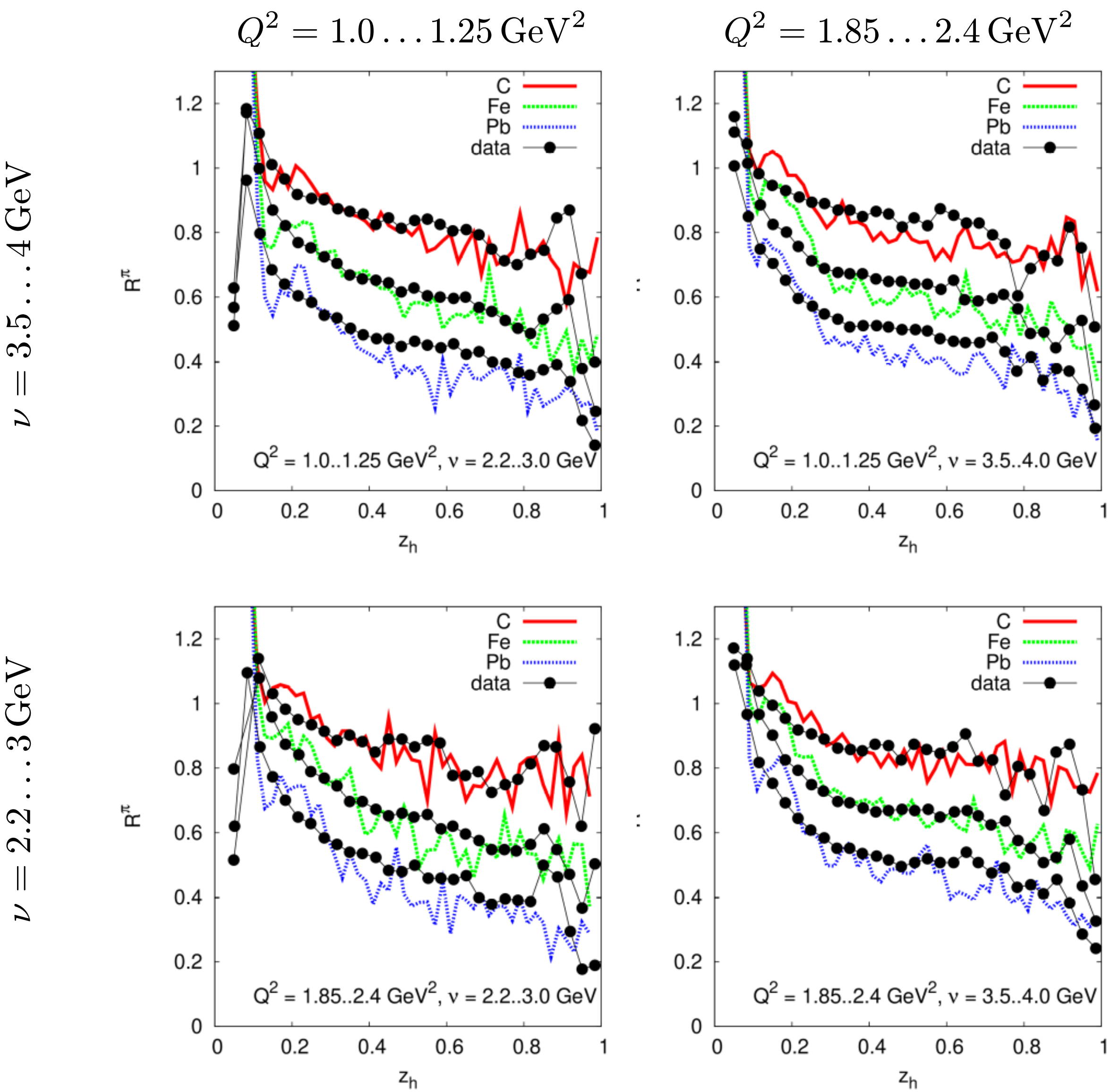
here: averaged times

in code: individual times

Hermes@27: A.Airapetian et al., NPB780(2007)1



CLAS@5, π⁺ : selected (ν,Q²) bins



Data:

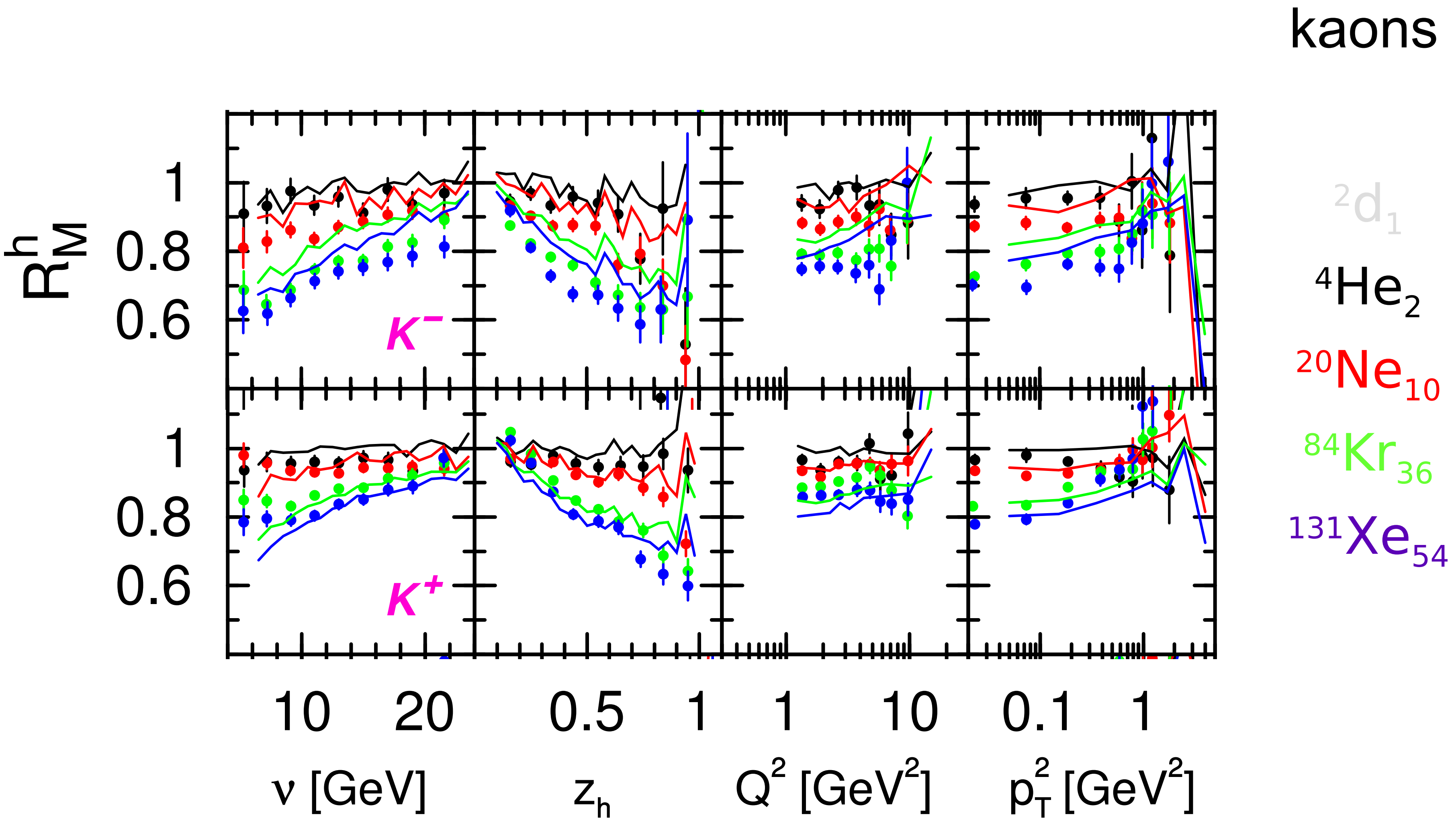
- CLAS preliminary
- no error bars shown

Calculations:

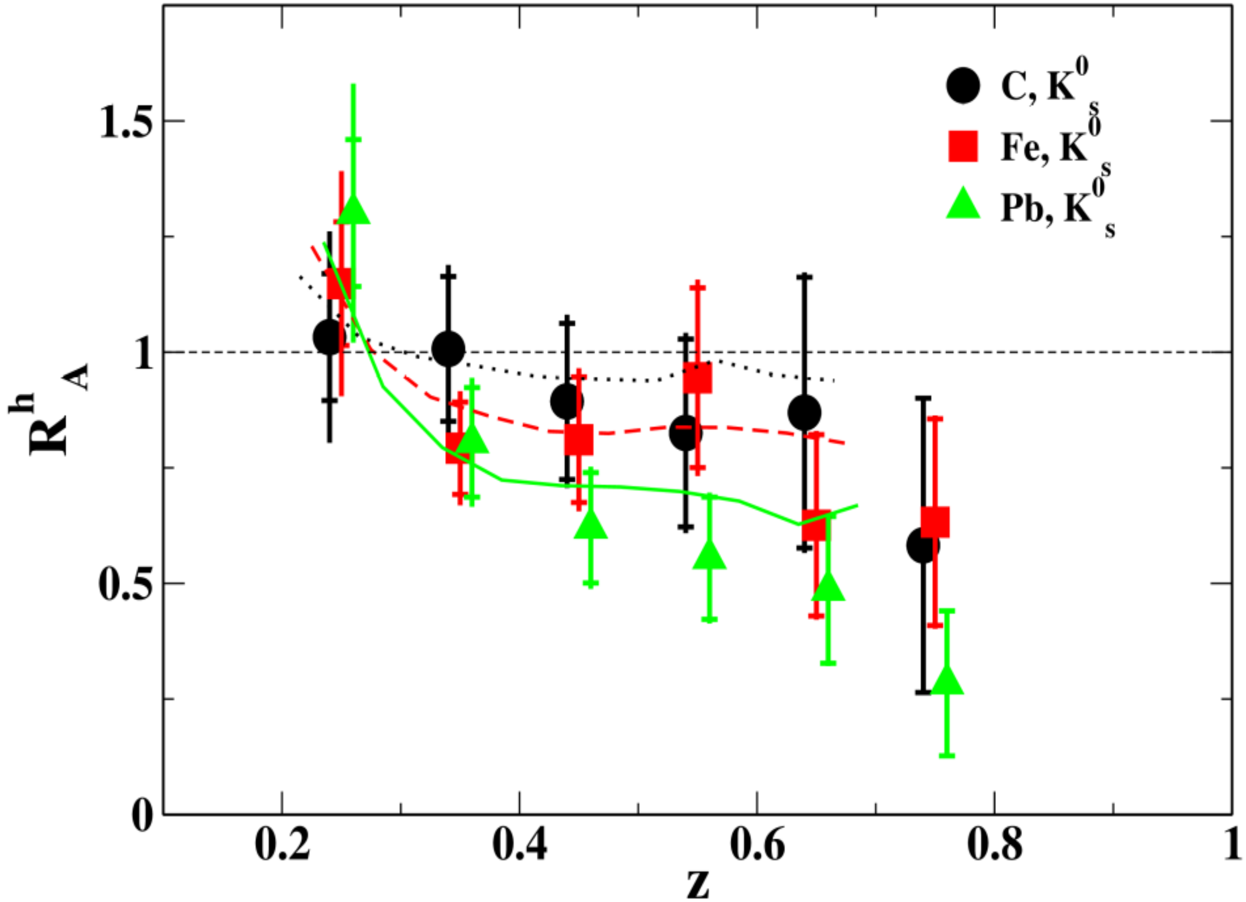
- not tuned !!!
- no Fermi Motion (W<2 GeV possible)
- no potentials

As good as at higher energies !

Hermes@27: A.Airapetian et al., NPB780(2007)1



CLAS@5, $K^0{}_s$: A.Daniel et al., PLB 706(2011) 26



$K^0 : d\bar{s}$

$K^+ : u\bar{s}$

$\overline{K^0} : \bar{d}s$

$K^- : \bar{u}s$

$$K^0 = \begin{cases} 50\% & K^0_L \\ 50\% & K^0_S \end{cases}$$

Predictions: K.G.,U.Mosel, NPA801(2008)68

Conclusions

- GiBUU (as all other transport models):
  - leading vs. non-leading hadrons
- e+A:
  - Hermes & EMC: great combination of energies
  - (pre-)hadron interaction increases linearly
    (not conclusive about CT effects)
  - relevant for JLAB@5, JLAB@12 and EIC

pi^0 analysis by T. Mineeva

K. Gallmeister, U. Mosel
``Time Dependent Hadronization via HERMES and EMC Data Consistency''
Nucl. Phys. A **801**(2008) 68

K. Gallmeister, T. Falter
``Space-time picture of fragmentation in PYTHIA/JETSET for HERMES and RHIC''
Phys. Lett. B **630** (2005) 40